Computational Coverage of Type Logical Grammar: The Montague Test

Glyn Morrill • Oriol Valentín

Abstract It is nearly half a century since Montague made his contributions to the field of logical semantics. In this time, computational linguistics has taken an almost entirely statistical turn and mainstream linguistics has adopted an almost entirely non-formal methodology. But in a minority approach reaching back before the linguistic revolution, and to the origins of computing, type logical grammar (TLG) has continued championing the flags of symbolic computation and logical rigor in discrete grammar. In this paper, we aim to concretise a measure of progress for computational grammar in the form of the *Montague Test*. This is the challenge of providing a computational cover grammar of the Montague fragment. We formulate this Montague Test and show how the challenge is met by the type logical parser/theorem-prover CatLog2.

Keywords Montague semantics · Montague grammar · categorial grammar · type logical grammar · computational grammar · semantic parsing · parsing as deduction · parsing/theorem-proving

G. Morrill, Universitat Politècnica de Catalunya, http://www.cs.upc.edu/~morrill/
O. Valentín, Universitat Politècnica de Catalunya, oriol.valentin@gmail.com

In Christopher Piñón (ed.), Empirical Issues in Syntax and Semantics 11, 141–170. Paris: CSSP.
http://www.cssp.cnrs.fr/eiss11/

1 Introduction

Perhaps nobody does Montague semantics anymore, or perhaps everybody does Montague semantics now and it has become a part of the scenery. Around 1970, Richard Montague wrote three papers, "Universal grammar" (Montague 1970b), "English as a formal language" (Montague 1970a), and "The proper treatment of quantification in ordinary English" (Montague 1973), which overturned the prevailing view that natural language semantics was too ephemeral to be formalised. The third paper, especially, introduced lambda calculus and higher-order intensional logic for semantic representation by presenting a formal fragment of English with a translation into logic.

Montague's approach was first popularised in the textbook Dowty et al. 1981. Since then, linguistics has become infused with Montague semantics starting with journals such as *Linguistics and Philosophy* and conferences such as the Amsterdam Colloquium, and spreading out in such a way that today there is an extensive interdisciplinary field of formal semantics based on lambda calculus and type logic. It is not that nobody does Montague semantics anymore, it is that now Montague semantics is taken for granted by many.

If you don't know where you have come from, you don't know where you are going. How can we be sure we are making progress? Here, in relation to Montague semantics, we propose as an exercise of intermediate difficulty, as a health check on approaches, the *Montague Test*, which is to provide a computational cover grammar of the Montague fragment as represented by the example sentences of Dowty et al. 1981:chap. 7.

Our broad concern is whether linguistics, rather than building on the achievements of the past and consolidating them, is rather in danger of drifting from trend to trend or lurching from fashion to fashion, in an aleatory or even cyclic fashion. Linguistics has its scholarly roots in the arts and humanities and from such origins a certain tendency to fantasia and self-proclamation persists. Perhaps this headiness partially explains why linguistics has remained a novice science while, for example, biology and computational biology have gone from strength to strength. Our plea here is that before a linguistic approach is deamed the new revolution, it proves its credentials by providing a computational cover grammar of the 50 years old Montague fragment.

In providing a computational cover grammar, we semantically parse the sentences provided with analysis trees in Dowty et al. 1981:chap. 7, assigning them logical translations "corresponding" to those given there, and distinguishing the same readings with comparable truth conditions. This minicorpus, which includes quantification, intensionality and some coordination and anaphora, is as follows:[1]

(7-7) **John walks** $walk'(j)$

[1]The reference numbers are taken directly from Dowty et al. 1981:chap. 7. Observe that the minicorpus preserves Montague's practice of assigning raised types to extensional verbs for uniformity with intensional verbs.

(7-16) **every man talks** $\forall x[man'(x) \to talk'(x)]$

(7-19) **the fish walks** $\exists y[\forall x[fish'(x) \leftrightarrow x = y] \wedge walk'(y)]$

(7-32) **every man walks or talks** $\forall y[man'(y) \to [walk'(y) \vee talk'(y)]]$

(7-34) **every man walks or every man talks**
$[\forall x[man'(x) \to walk'(x)] \vee \forall x[man'(x) \to talk'(x)]]$

(7-39) **a woman walks and she talks**
$\exists x[woman'(x) \wedge [walk'(x) \wedge talk'(x)]]$

(7-43, 45) **John believes that a fish walks**
$believe'(j, {}^\wedge\exists x[fish'(x) \wedge walk'(x)])$
$\exists x[fish'(x) \wedge believe'(j, {}^\wedge[walk'(x)])]$

(7-48, 49, 52) **every man believes that a fish walks**
$\exists x[fish'(x) \wedge \forall y[man'(y) \to believe'(y, {}^\wedge[walk'(x)])]]$
$\forall y[man'(y) \to \exists x[fish'(x) \wedge believe'(y, {}^\wedge[walk'(x)])]]$
$\forall y[man'(y) \to believe'(y, {}^\wedge[\exists x[fish'(x) \wedge walk'(x)]])]$

(7-57) **every fish such that it walks talks**
$\forall x[[fish'(x) \wedge walk'(x)] \to talk'(x)]$

(7-60, 62) **John seeks a unicorn**
$try'(j, {}^\wedge[find'({}^\wedge\lambda P\exists x[unicorn'(x) \wedge [{}^\vee P](x)])])$
$try'(j, {}^\wedge\lambda z[\exists x[unicorn'(x) \wedge [find'({}^\wedge\lambda P[[{}^\vee P](z)])(j)]]])$

(7-73) **John is Bill** $j = b$

(7-76) **John is a man** $man'(j)$

(7-83) **necessarily John walks** $\Box[walk'(j)]$

(7-86) **John walks slowly** $slowly'({}^\wedge walk')(j)$

(7-91) **John tries to walk** $try'({}^\wedge walk')(j)$

(7-94) **John tries to catch a fish and eat it**
$try'(j, {}^\wedge\lambda y\exists x[fish'(x) \wedge$
$[catch'({}^\wedge\lambda P[[{}^\vee P](y)])(x)) \wedge eat'({}^\wedge\lambda P[[{}^\vee P](y)])(x))]])$

	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	limited contr. & weak.
primary	$/$ $\backslash$ $\bullet$ I	$\uparrow$ $\downarrow$ $\odot$ J	$\&$ $\oplus$	$\bigwedge$ $\bigvee$	$\Box$ $\Diamond$	$[\,]^{-1}$ $\langle\rangle$	$!$ $?$	$\vert$ W
sem. inactive variants	•— —∘ ∘— —• ◐ ◑	↑̇ ↓̊ ↑̊ ↓̇ ◓ ◒	$\sqcap$ $\sqcup$	$\forall$ $\exists$	■ ♦			
det. synth.	$\triangleleft^{-1}$ $\triangleright^{-1}$ $\triangleleft$ $\triangleright$	ˇ ˆ						diff.
nondet. synth.	$\div$ $\circ$	$\stackrel{\wedge}{=}$ $\stackrel{\vee}{=}$ $\diamond$						$-$

Table 1 Categorial connectives

(7-98) **John finds a unicorn**

$\exists x[\mathit{unicorn}'(x) \wedge [\mathit{find}'(^\wedge\lambda P[[^\vee P](x)])(j)]]$

(7-105) **every man such that he loves a woman loses her**

$\exists y[\mathit{woman}'(y) \wedge \forall x[[\mathit{man}'(x) \wedge \mathit{love}'(^\wedge\lambda P([[^\vee P](y)])(x)] \rightarrow$
$\mathit{lose}'(^\wedge\lambda P([[^\vee P](y)])(x)]]$

(7-110) **John walks in a park**

$\exists x[\mathit{park}'(x) \wedge \mathit{in}'(^\wedge\lambda P[[^\vee P](x)])(^\wedge\mathit{walk}')(j)]$

(7-116, 118) **every man doesn't walk**

$\neg\forall x[\mathit{man}'(x) \rightarrow \mathit{walk}'(x)]$
$\forall x[\mathit{man}'(x) \rightarrow \neg\mathit{walk}'(x)]$

2 Type Logical Grammar

Type logical grammar (TLG) is a categorial theory of syntax and semantics in which words and expressions are classified by logical types. TLG is expounded in Moortgat 1988, 1997, Morrill 1994, 2011, Carpenter 1997, Jäger 2005, Moot & Retoré 2012. The logical types form an intuitionistic sublinear logic and their rules are universal; a grammar comprises just a lexicon classifying basic expressions. TLG is thus a purely lexical formalism.

A *sign* $\alpha: A: \phi$ consists of a *prosodic form* α, a *syntactic type* A, and a *semantic form* ϕ. A *prosodic sort map* s maps syntactic types to prosodic sorts which are the number of points of discontinuity of expressions of that type; a *semantic type map* T maps syntactic types to semantic types which are essentially formulas of intuitionistic propositional logic/types of lambda calculus under the Curry-Howard correspondence. In a sign $\alpha: A: \phi$, α must be of prosodic sort $s(A)$ and ϕ must be of semantic type $T(A)$.

The categorial connectives of our type logical grammar are as shown in table 1. They comprise the primary connectives, in the first row, semantically inactive variants, in the second row, and deterministic (unary) and nondeterministic (binary) defined connectives in the third and fourth rows.

Regarding the primary connectives, the displacement connectives (Morrill et al. 2011) are made up of the continuous (Lambek) and discontinuous multiplicatives. Then there are additives (Morrill 1990a), quantifiers (Morrill 1994), normal modalities (Morrill 1990b, Moortgat 1997), bracket modalities (Morrill 1992, Moortgat 1996), exponentials (Morrill & Valentín 2015a), limited contraction (Jäger 2005) and limited weakening (Morrill & Valentín 2014b).

The semantically inactive secondary connectives are made up of semantically inactive multiplicatives (Morrill & Valentín 2014b), additives (Morrill 1994), quantifiers (Morrill 1994), and normal modalities (Hepple 1990, Moortgat 1997). The deterministic secondary connectives are made up of the unary connectives projection and injection (Morrill et al. 2009) and split and bridge (Morrill & Merenciano 1996), and the nondeterministic secondary connectives are made up of concatenative binary connectives of division and product and discontinuous binary connectives of extraction, infixation and product (Morrill et al. 2011). At the bottom right is a metalogical ("negation as failure") connective of difference (Morrill & Valentín 2014a).

A lexicon consists of a set of (lexical) signs. Our lexicon for the Montague fragment is as follows; rules for connectives used in the fragment are given in the Appendix:

a : $\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\exists C[(A\ C) \wedge (B\ C)]$

and : $\blacksquare\forall f((\blacksquare ?Sf\backslash[]^{-1}[]^{-1}Sf)/\blacksquare Sf) : (\Phi^{n+}\ o\ and)$

and : $\blacksquare\forall a\forall f((\blacksquare ?(\langle\rangle Na\backslash Sf)\backslash[]^{-1}[]^{-1}(\langle\rangle Na\backslash Sf))/\blacksquare(\langle\rangle Na\backslash Sf))$:
$(\Phi^{n+}\ (s\ o)\ and)$

believes : $\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\square Sf)) : {}^\wedge\lambda A\lambda B((^\vee believe\ A)\ B)$

bill : $\blacksquare Nt(s(m)) : b$

catch : $\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^\wedge\lambda A\lambda B((^\vee catch\ A)\ B)$

doesnt : $\blacksquare\forall g\forall a((Sg{\uparrow}((\langle\rangle Na\backslash Sf)/(\langle\rangle Na\backslash Sb))){\downarrow}Sg) : \lambda A\neg(A\ \lambda B\lambda C(B\ C))$

eat : $\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^\wedge\lambda A\lambda B((^\vee eat\ A)\ B)$

every : $\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)]$

finds : $\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : {}^\wedge\lambda A\lambda B((^\vee find\ A)\ B)$

fish : $\square CNs(n) : fish$

he : $\blacksquare[]^{-1}\forall g((\blacksquare Sg|\blacksquare Nt(s(m)))/(\langle\rangle Nt(s(m))\backslash Sg)) : \lambda AA$

her : $\blacksquare\forall g\forall a(((\langle\rangle Na\backslash Sg){\uparrow}\blacksquare Nt(s(f))){\downarrow}(\blacksquare(\langle\rangle Na\backslash Sg)|\blacksquare Nt(s(f)))) : \lambda AA$

in : $\square(\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))/\exists aNa) : {}^\wedge\lambda A\lambda B\lambda C((^\vee in\ A)\ (B\ C))$

is : $\blacksquare((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg))-I)))$:
$\lambda A\lambda B(A \rightarrow C.[B = C]; D.((D\ \lambda E[E = B])\ B))$

it : $\blacksquare\forall f\forall a(((\langle\rangle Na\backslash Sf){\uparrow}\blacksquare Nt(s(n))){\downarrow}(\blacksquare(\langle\rangle Na\backslash Sf)|\blacksquare Nt(s(n)))) : \lambda AA$

it : $\blacksquare[]^{-1}\forall f((\blacksquare Sf|\blacksquare Nt(s(n)))/(\langle\rangle Nt(s(n))\backslash Sf)) : \lambda AA$

john : $\blacksquare Nt(s(m)) : j$

loses : $\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : {}^\wedge\lambda A\lambda B((^\vee lose\ A)\ B)$

loves : $\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : {}^\wedge\lambda A\lambda B((^\vee love\ A)\ B)$

man : $\square CNs(m) : man$

necessarily : $\blacksquare(SA/\square SA) : Nec$

or : $\blacksquare\forall f((\blacksquare ?Sf\backslash[]^{-1}[]^{-1}Sf)/\blacksquare Sf) : (\Phi^{n+}\ o\ or)$

or : $\blacksquare\forall a\forall f((\blacksquare ?(\langle\rangle Na\backslash Sf)\backslash[]^{-1}[]^{-1}(\langle\rangle Na\backslash Sf))/\blacksquare(\langle\rangle Na\backslash Sf))$:
$(\Phi^{n+}\ (s\ o)\ or)$

or : $\blacksquare\forall f((\blacksquare ?(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf))\backslash[]^{-1}[]^{-1}(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf)))/$
$\blacksquare(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf))) : (\Phi^{n+}\ (s\ o)\ or)$

park : $\square CNs(n) : park$

seeks : $\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\square\forall a\forall f(((Na\backslash Sf)/\exists bNb)\backslash(Na\backslash Sf)))$:
${}^\wedge\lambda A\lambda B((^\vee try\ {}^\wedge((^\vee A\ {}^\vee find)\ B))\ B)$

she : $\blacksquare[]^{-1}\forall g((\blacksquare Sg|\blacksquare Nt(s(f)))/(\langle\rangle Nt(s(f))\backslash Sg)) : \lambda AA$

slowly : $\square\forall a\forall f(\square(\langle\rangle Na\backslash Sf)\backslash(\langle\rangle\square Na\backslash Sf))$: $\hat{}\lambda A\lambda B(\check{}slowly\ \hat{}(\check{}A\ \check{}B))$
such+that : $\blacksquare\forall n((CNn\backslash CNn)/(Sf|\blacksquare Nt(n)))$: $\lambda A\lambda B\lambda C[(B\ C)\wedge(A\ C)]$
talks : $\square(\langle\rangle\exists gNt(s(g))\backslash Sf)$: $\hat{}\lambda A(\check{}talk\ A)$
that : $\blacksquare(CPthat/\square Sf)$: λAA
the : $\blacksquare\forall n(Nt(n)/CNn)$: ι
to : $\blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash Si)/(\langle\rangle Nn\backslash Sb)))$: λAA
tries : $\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\square(\langle\rangle\exists gNt(s(g))\backslash Si))$:
$\hat{}\lambda A\lambda B((\check{}try\ \hat{}(\check{}A\ B))\ B)$
unicorn : $\square CNs(n)$: *unicorn*
walk : $\square(\langle\rangle\exists aNa\backslash Sb)$: $\hat{}\lambda A(\check{}walk\ A)$
walks : $\square(\langle\rangle\exists gNt(s(g))\backslash Sf)$: $\hat{}\lambda A(\check{}walk\ A)$
woman : $\square CNs(f)$: *woman*

3 Performing the Montague Test

CatLog2 is a type logical parser/theorem prover with a web interface at `http://www.cs.upc.edu/~morrill/CatLog/CatLog2/index.php`. It:

- comprises 6000 lines of prolog
- has 20 primitive categorial connectives, 29 defined connectives, and 1 metalogical connective: a total of 50 connectives
- has typically 2 rules for each connective: a rule of use and a rule of proof: roughly $50\times 2=100$ rules
- uses backward chaining sequent proof search and uses *focusing* (Andreoli 1992); for the focused rules—about half of them—for a binary connective there are 4 cases of "polarity": $+/+$, $+/-$, $-/+$, $-/-$: $50+50\times 4=$ a total of about 250 rules

At CSSP in Paris on 9 October 2015, the Montague Test was performed by CatLog2 version "gmontague" with input in the following format; note that currently it is necessary to give syntactic domains in the input to CatLog2 (though these play no role in Montague's grammar):

str(dwp('(7-7)'), [b([john]), walks], s(f)).
str(dwp('(7-16)'), [b([every, man]), talks], s(f)).
str(dwp('(7-19)'), [b([the, fish]), walks], s(f)).
str(dwp('(7-32)'), [b([every, man]), b([b([walks, or, talks])])], s(f)).

str(dwp('(7-34)'), [b([b([b([every, man]), walks, or, b([every, man]), talks])])], s(f)).
str(dwp('(7-39)'), [b([b([b([a, woman]), walks, and, b([she]), talks])])], s(f)).
str(dwp('(7-43, 45)'), [b([john]), believes, that, b([a, fish]), walks], s(f)).
str(dwp('(7-48, 49, 52)'), [b([every, man]), believes, that, b([a, fish]), walks], s(f)).
str(dwp('(7-57)'), [b([every, fish, such, that, b([it]), walks]), talks], s(f)).
str(dwp('(7-60, 62)'), [b([john]), seeks, a, unicorn], s(f)).
str(dwp('(7-73)'), [b([john]), is, bill], s(f)).
str(dwp('(7-76)'), [b([john]), is, a, man], s(f)).
str(dwp('(7-83)'), [necessarily, b([john]), walks], s(f)).
str(dwp('(7-86)'), [b([john]), walks, slowly], s(f)).
str(dwp('(7-91)'), [b([john]), tries, to, walk], s(f)).
str(dwp('(7-94)'), [b([john]), tries, to, b([b([catch, a, fish, and, eat, it])])], s(f)).
str(dwp('(7-98)'), [b([john]), finds, a, unicorn], s(f)).
str(dwp('(7-105)'), [b([every, man, such, that, b([he]), loves, a, woman]), loses, her], s(f)).
str(dwp('(7-110)'), [b([john]), walks, in, a, park], s(f)).
str(dwp('(7-116, 118)'), [b([every, man]), doesnt, walk], s(f)).

The LaTeX output generated was as follows. Each item comes in the form of its identifier and the prosodic form of its input, followed by each semantically labelled sequent that results from lexical lookup. Where there is a derivation or derivations for a sequent, these appear in figures with the semantic forms delivered by the analysis in the main text. CatLog2 observes the proof search discipline of *focusing* (Andreoli 1992, Morrill & Valentín 2015b): in the derivations the focused types are boxed, which means that when a complex type in a conclusion is boxed, it is the active type of the inference. For reasons of space, some derivations are omitted.

(dwp((7-7))) [**john**]+**walks** : Sf

$$[\blacksquare Nt(s(m)) : j], \square(\langle\rangle\exists g Nt(s(g))\backslash Sf) : \hat{}\lambda A(\check{}walk\ A) \ \Rightarrow\ Sf$$

$$
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\overline{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\ \blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists g Nt(s(g))}}\ \exists R}{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle \exists g Nt(s(g))}}\ \langle\rangle R \qquad \overline{\boxed{Sf} \Rightarrow Sf}}{[\blacksquare Nt(s(m))], \boxed{\langle\rangle \exists g Nt(s(g)) \backslash Sf} \Rightarrow Sf}\ \backslash L}{[\blacksquare Nt(s(m))], \boxed{\square(\langle\rangle \exists g Nt(s(g)) \backslash Sf)} \Rightarrow Sf}\ \square L
$$

Figure 1 Derivation of (dwp((7-7)))

For the derivation, see figure 1.

$(\check{}walk\ j)$

(dwp((7-16))) [**every**+**man**]+**talks** : Sf

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(m) : man], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}talk\ D) \Rightarrow Sf$

For the derivation, see figure 2.

$\forall C[(\check{}man\ C) \rightarrow (\check{}talk\ C)]$

(dwp((7-19))) [**the**+**fish**]+**walks** : Sf

$[\blacksquare\forall n(Nt(n)/CNn) : \iota, \square CNs(n) : fish], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda A(\check{}walk\ A) \Rightarrow Sf$

(Derivation omitted)

$(\check{}walk\ (\iota\ \check{}fish))$

(dwp((7-32))) [**every**+**man**]+[[**walks**+**or**+**talks**]] : Sf

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(m) : man], [[\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall f((\blacksquare ?Sf\backslash[]^{-1}[]^{-1}Sf)/\blacksquare Sf) : (\Phi^{n+}\ o\ or), \square(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda E(\check{}talk\ E)]] \Rightarrow Sf$

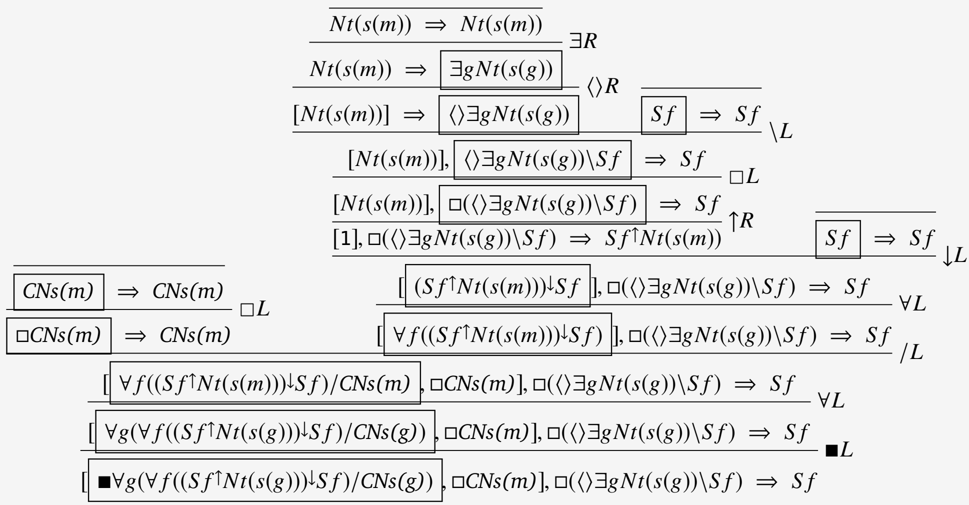

Figure 2 Derivation of (dwp((7-16)))

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \to (B\ C)],$
$\square CNs(m) : man], [[\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall a\forall f((\blacksquare?(\langle\rangle Na\backslash Sf)\backslash[]^{-1}[]^{-1}(\langle\rangle Na\backslash Sf))/\blacksquare(\langle\rangle Na\backslash Sf)) : (\Phi^{n+}\ (s\ o)\ or),$
$\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda E(\check{}talk\ E)]] \Rightarrow Sf$

(Derivation omitted)

$\forall C[(\check{}man\ C) \to [(\check{}walk\ C) \vee (\check{}talk\ C)]]$

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \to (B\ C)],$
$\square CNs(m) : man], [[\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall f((\blacksquare?(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf))\backslash[]^{-1}[]^{-1}(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf)))/$
$\blacksquare(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf))) : (\Phi^{n+}\ (s\ o)\ or), \square(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda E(\check{}talk\ E)]] \Rightarrow Sf$

(dwp((7-34))) [[[**every**+**man**]+**walks**+**or**+[**every**+**man**]+**talks**]] : Sf

$[[[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \to (B\ C)],$
$\square CNs(m) : man], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall f((\blacksquare?Sf\backslash[]^{-1}[]^{-1}Sf)/\blacksquare Sf) : (\Phi^{n+}\ o\ or),$
$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda E\lambda F\forall G[(E\ G) \to (F\ G)],$
$\square CNs(m) : man], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda H(\check{}talk\ H)]] \Rightarrow Sf$

(Derivation omitted)

$[\forall H[(\check{}man\ H) \rightarrow (\check{}walk\ H)] \vee \forall C[(\check{}man\ C) \rightarrow (\check{}talk\ C)]]$

$[[[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(m) : man], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall a\forall f((\blacksquare?(\langle\rangle Na\backslash Sf)\backslash[]^{-1}[]^{-1}(\langle\rangle Na\backslash Sf))/\blacksquare(\langle\rangle Na\backslash Sf)) : (\Phi^{n+}\ (s\ o)\ or),$
$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda E\lambda F\forall G[(E\ G) \rightarrow (F\ G)],$
$\square CNs(m) : man], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda H(\check{}talk\ H)]] \ \Rightarrow\ Sf$

$[[[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(m) : man], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall f((\blacksquare?(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf))\backslash[]^{-1}[]^{-1}(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf)))/$
$\blacksquare(Sf/(\langle\rangle\exists gNt(s(g))\backslash Sf))) : (\Phi^{n+}\ (s\ o)\ or),$
$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda E\lambda F\forall G[(E\ G) \rightarrow (F\ G)],$
$\square CNs(m) : man], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda H(\check{}talk\ H)]] \ \Rightarrow\ Sf$

(dwp((7-39))) [[[**a**+**woman**]+**walks**+**and**+[**she**]+**talks**]] : Sf

$[[[\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\exists C[(A\ C) \wedge (B\ C)],$
$\square CNs(f) : woman], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall f((\blacksquare?Sf\backslash[]^{-1}[]^{-1}Sf)/\blacksquare Sf) : (\Phi^{n+}\ o\ and),$
$[\blacksquare[]^{-1}\forall g((\blacksquare Sg|\blacksquare Nt(s(f)))/(\langle\rangle Nt(s(f))\backslash Sg)) : \lambda EE],$
$\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda F(\check{}talk\ F)]] \ \Rightarrow\ Sf$

(Derivation omitted)

$\exists C[(\check{}woman\ C) \wedge [(\check{}walk\ C) \wedge (\check{}talk\ C)]]$

$[[[\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\exists C[(A\ C) \wedge (B\ C)],$
$\square CNs(f) : woman], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda D(\check{}walk\ D),$
$\blacksquare\forall a\forall f((\blacksquare?(\langle\rangle Na\backslash Sf)\backslash[]^{-1}[]^{-1}(\langle\rangle Na\backslash Sf))/\blacksquare(\langle\rangle Na\backslash Sf)) :$
$(\Phi^{n+}\ (s\ o)\ and),$
$[\blacksquare[]^{-1}\forall g((\blacksquare Sg|\blacksquare Nt(s(f)))/(\langle\rangle Nt(s(f))\backslash Sg)) : \lambda EE],$
$\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda F(\check{}talk\ F)]] \ \Rightarrow\ Sf$

(dwp((7-43, 45))) [**john**]+**believes**+**that**+[**a**+**fish**]+**walks** : Sf

$[\blacksquare Nt(s(m)) : j], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\square Sf)) :$
$\hat{}\lambda A\lambda B((\check{}believe\ A)\ B), \blacksquare(CPthat/\square Sf) : \lambda CC,$

$[\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda D\lambda E\exists F[(D\ F) \wedge (E\ F)],$
$\square CNs(n) : fish], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda G(\check{}walk\ G) \;\Rightarrow\; Sf$

For the derivation, see figure 3.

$\exists C[(\check{}fish\ C) \wedge ((\check{}believe\ \hat{}(\check{}walk\ C))\ j)]$

For the derivation, see figure 4.

$((\check{}believe\ \hat{}\exists F[(\check{}fish\ F) \wedge (\check{}walk\ F))])\ j)$

(dwp((7-48, 49, 52))) [**every**+**man**]+**believes**+**that**+[**a**+**fish**]+**walks** : Sf

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(m) : man], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\square Sf)) :$
$\hat{}\lambda D\lambda E((\check{}believe\ D)\ E), \blacksquare(CPthat/\square Sf) : \lambda FF,$
$[\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda G\lambda H\exists I[(G\ I) \wedge (H\ I)],$
$\square CNs(n) : fish], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda J(\check{}walk\ J) \;\Rightarrow\; Sf$

(Derivation omitted)

$\exists C[(\check{}fish\ C) \wedge \forall G[(\check{}man\ G) \rightarrow ((\check{}believe\ \hat{}(\check{}walk\ C))\ G)]]$

(Derivation omitted)

$\forall C[(\check{}man\ C) \rightarrow \exists G[(\check{}fish\ G) \wedge ((\check{}believe\ \hat{}(\check{}walk\ G))\ C)]]$

(Derivation omitted)

$\forall C[(\check{}man\ C) \rightarrow ((\check{}believe\ \hat{}\exists J[(\check{}fish\ J) \wedge (\check{}walk\ J)])\ C)]$

(dwp((7-57))) [**every**+**fish**+**such**+**that**+[**it**]+**walks**]+**talks** : Sf

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(n) : fish, \blacksquare\forall n((CNn\backslash CNn)/(Sf|\blacksquare Nt(n))) : \lambda D\lambda E\lambda F[(E\ F) \wedge (D\ F)],$
$[\blacksquare\forall f\forall a(((\langle\rangle Na\backslash Sf){\uparrow}\blacksquare Nt(s(n))){\downarrow}(\blacksquare(\langle\rangle Na\backslash Sf)|\blacksquare Nt(s(n)))) : \lambda GG],$
$\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda H(\check{}walk\ H)], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda I(\check{}talk\ I) \;\Rightarrow\; Sf$

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(n) : fish, \blacksquare\forall n((CNn\backslash CNn)/(Sf|\blacksquare Nt(n))) : \lambda D\lambda E\lambda F[(E\ F) \wedge (D\ F)],$
$[\blacksquare[\,]^{-1}\forall f((\blacksquare Sf|\blacksquare Nt(s(n)))/(\langle\rangle Nt(s(n))\backslash Sf)) : \lambda GG],$

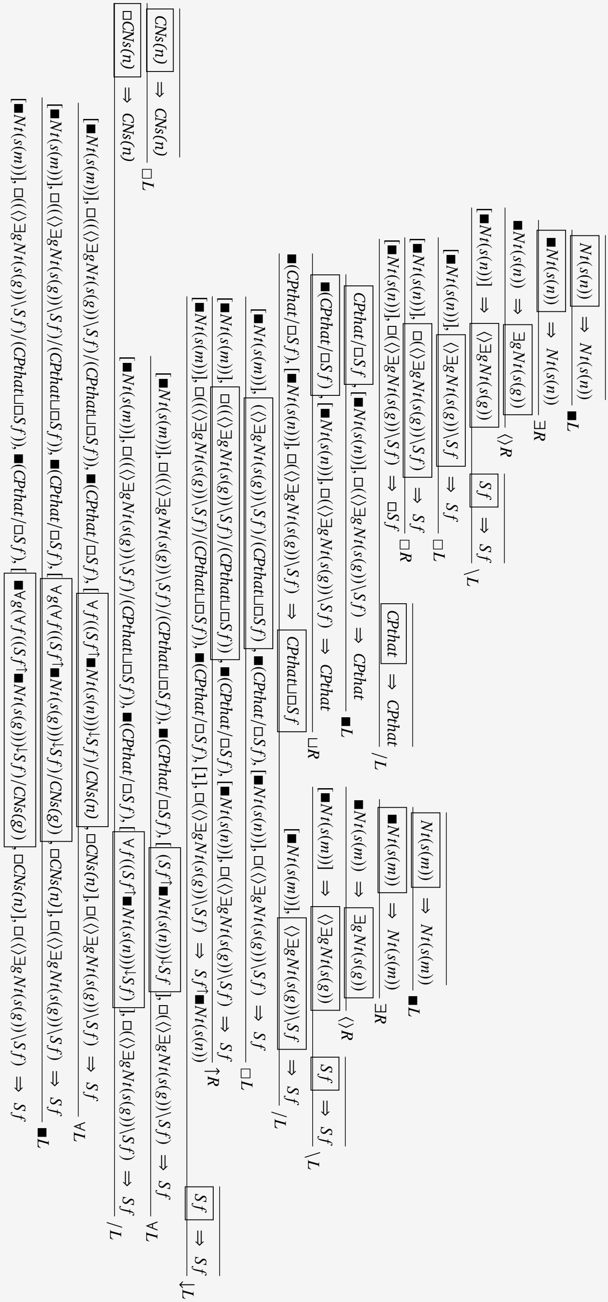

Figure 3 First derivation of (dwp((7-43, 45)))

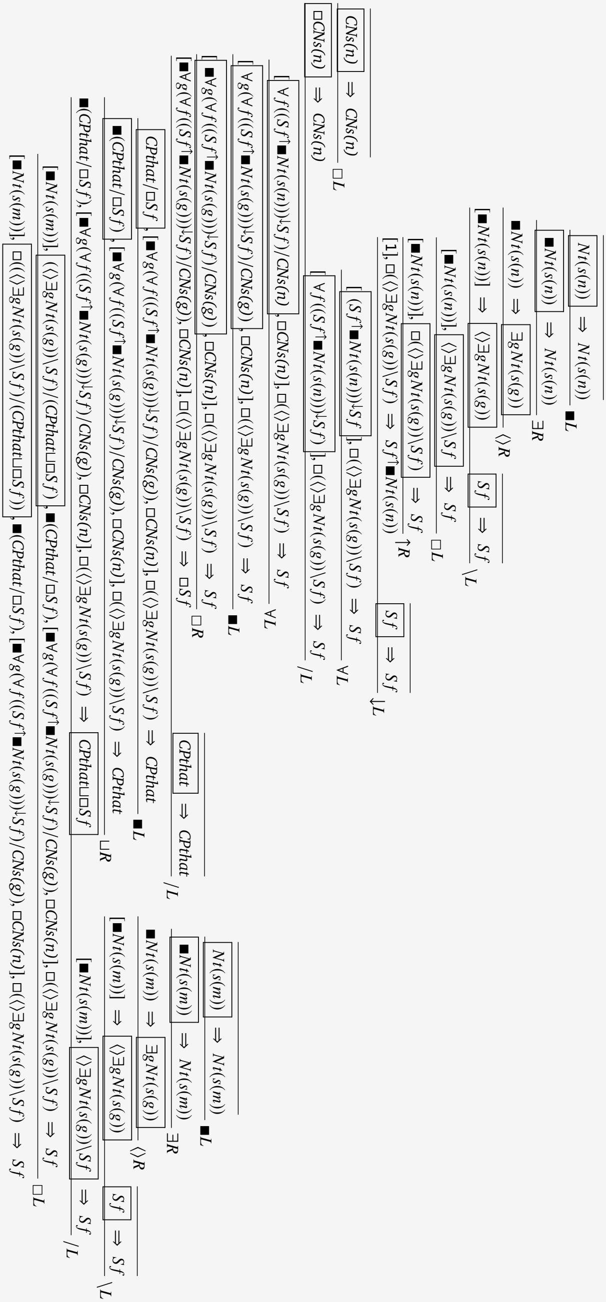

Figure 4 Second derivation of (dwp((7-43, 45)))

$\square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda H(\check{}walk\ H)], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda I(\check{}talk\ I) \ \Rightarrow\ Sf$

(Derivation omitted)

$\forall C[[(\check{}fish\ C) \wedge (\check{}walk\ C)] \rightarrow (\check{}talk\ C)]$

(dwp((7-60, 62))) [**john**]+**seeks**+**a**+**unicorn** : Sf

$[\blacksquare Nt(s(m)) : j], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\ \square\forall a\forall f(((Na\backslash Sf)/\exists bNb)\backslash(Na\backslash Sf))) : \hat{}\lambda A\lambda B((\check{}try\ \hat{}((\check{}A\ \check{}find)\ B))\ B), \blacksquare\forall g(\forall f((Sf\uparrow\blacksquare Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda C\lambda D\exists E[(C\ E) \wedge (D\ E)], \square CNs(n) : unicorn \ \Rightarrow\ Sf$

For the derivation, see figure 5.

$\exists C[(\check{}unicorn\ C) \wedge ((\check{}try\ \hat{}((\check{}find\ C)\ j))\ j)]$

For the derivation, see figure 6.

$((\check{}try\ \hat{}\exists G[(\check{}unicorn\ G) \wedge ((\check{}find\ G)\ j)])\ j)$

(dwp((7-73))) [**john**]+**is**+**bill** : Sf

$[\blacksquare Nt(s(m)) : j], \blacksquare((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg))-I))) : \lambda A\lambda B(A \rightarrow C.[B = C]; D.((D\ \lambda E[E = B])\ B)), \blacksquare Nt(s(m)) : b \ \Rightarrow\ Sf$

For the derivation, see figure 7.

$[j = b]$

(dwp((7-76))) [**john**]+**is**+**a**+**man** : Sf

$[\blacksquare Nt(s(m)) : j], \blacksquare((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg))-I))) : \lambda A\lambda B(A \rightarrow C.[B = C]; D.((D\ \lambda E[E = B])\ B)), \blacksquare\forall g(\forall f((Sf\uparrow\blacksquare Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda F\lambda G\exists H[(F\ H) \wedge (G\ H)], \square CNs(m) : man \ \Rightarrow\ Sf$

For the derivation, see figure 8.

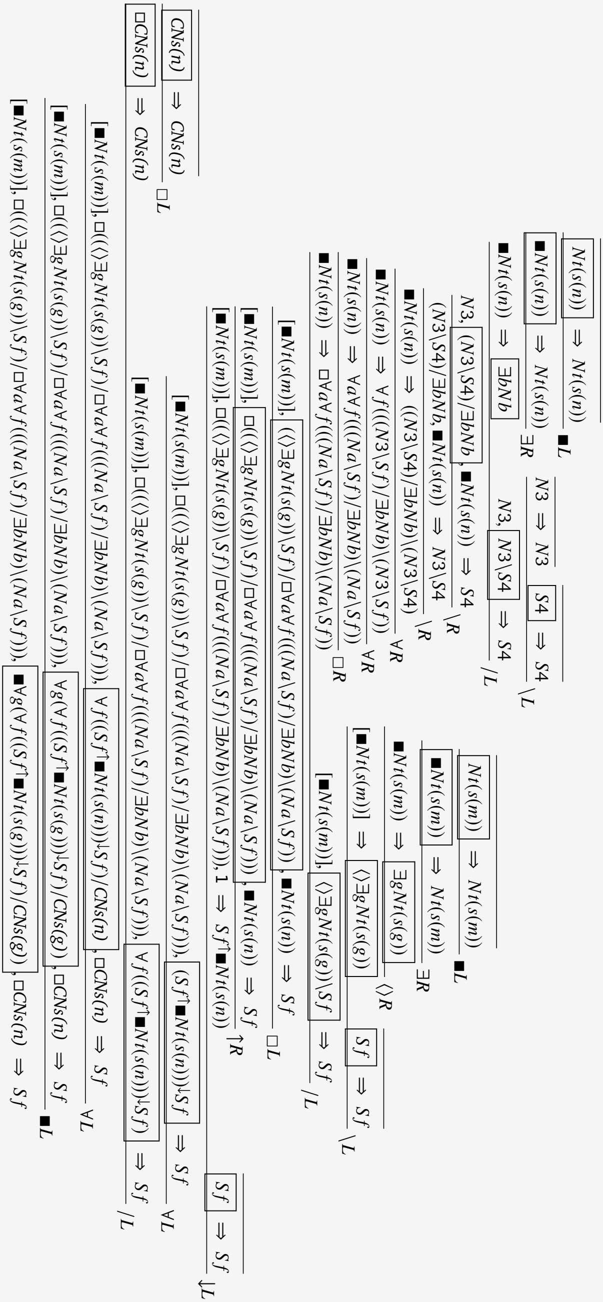

Figure 5 First derivation of (dwp((7-60, 62)))

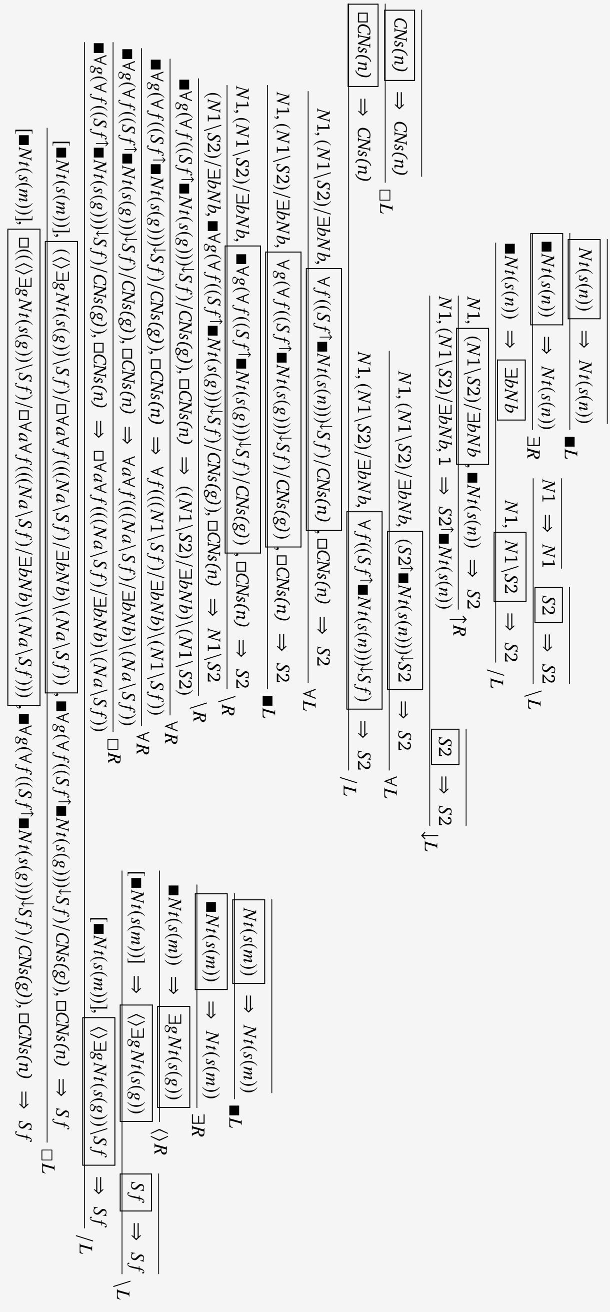

Figure 6 Second derivation of (dwp((7-60, 62)))

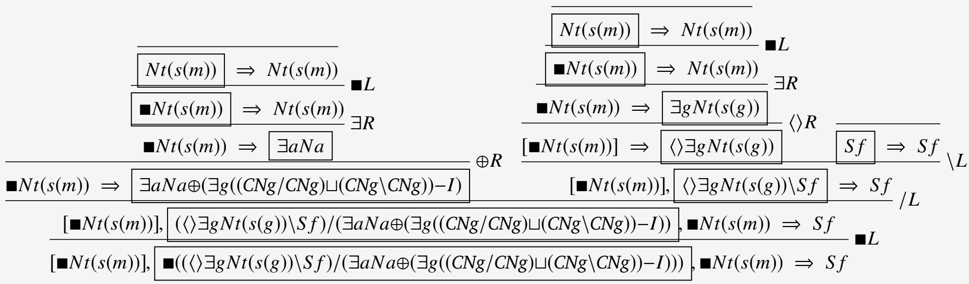

Figure 7 Derivation of (dwp((7-73)))

$\exists C[(\check{}man\ C) \wedge [j = C]]$

(dwp((7-83))) **necessarily+[john]+walks** : Sf

$\blacksquare(SA/\Box SA) : Nec, [\blacksquare Nt(s(m)) : j], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda B(\check{}walk\ B) \ \Rightarrow\ Sf$

(Derivation omitted)

$(Nec\ \hat{}((\check{}walk\ j))$

(dwp((7-86))) **[john]+walks+slowly** : Sf

$[\blacksquare Nt(s(m)) : j], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda A(\check{}walk\ A),$
$\Box\forall a\forall f(\Box(\langle\rangle Na\backslash Sf)\backslash(\langle\rangle\Box Na\backslash Sf)) : \hat{}\lambda B\lambda C(\check{}slowly\ \hat{}(\check{}B\ \check{}C)) \ \Rightarrow\ Sf$

(Derivation omitted)

$(\check{}slowly\ \hat{}(\check{}walk\ j))$

(dwp((7-91))) **[john]+tries+to+walk** : Sf

$[\blacksquare Nt(s(m)) : j], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\Box(\langle\rangle\exists gNt(s(g))\backslash Si)) :$
$\hat{}\lambda A\lambda B((\check{}try\ \hat{}(\check{}A\ B))\ B), \blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash Si)/(\langle\rangle Nn\backslash Sb))) :$
$\lambda CC, \Box(\langle\rangle\exists aNa\backslash Sb) : \hat{}\lambda D(\check{}walk\ D) \ \Rightarrow\ Sf$

(Derivation omitted)

$((\check{}try\ \hat{}(\check{}walk\ j))\ j)$

(dwp((7-94))) **[john]+tries+to+[[catch+a+fish+and+eat+it]]** : Sf

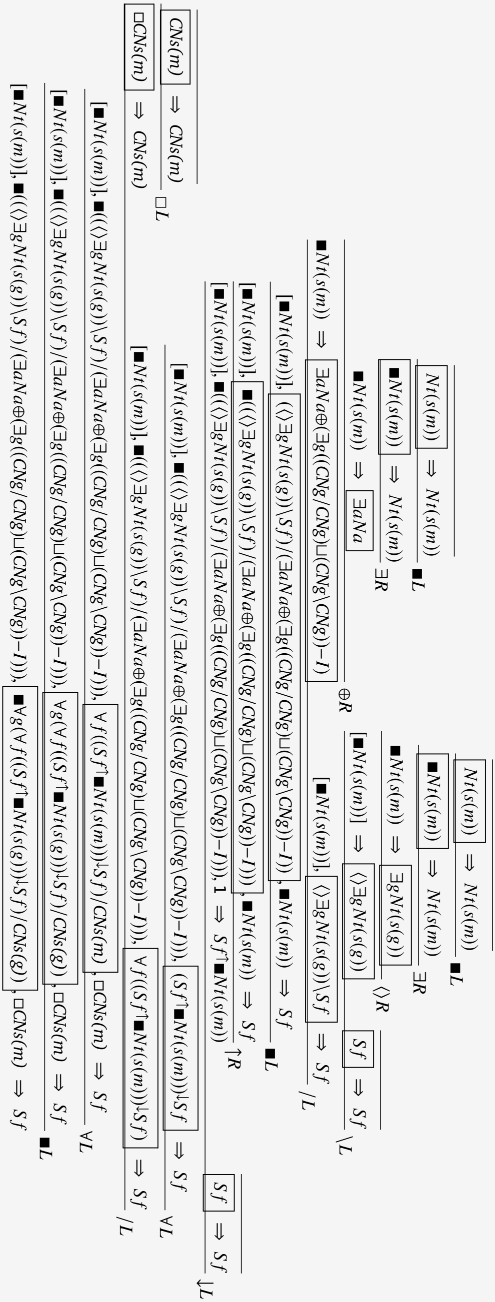

Figure 8 Derivation of (dwp((7-76)))

$$\begin{array}{l}
[\blacksquare Nt(s(m)) : j], \square(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\square(\langle\rangle\exists gNt(s(g))\backslash Si)) : \\
{}^{\wedge}\lambda A\lambda B(({}^{\vee}\mathit{try}\ {}^{\wedge}({}^{\vee}A\ B))\ B), \blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash Si)/(\langle\rangle Nn\backslash Sb))) : \\
\lambda CC, [[\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^{\wedge}\lambda D\lambda E(({}^{\vee}\mathit{catch}\ D)\ E), \\
\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda F\lambda G\exists H[(F\ H)\wedge(G\ H)], \\
\square CNs(n) : \mathit{fish}, \blacksquare\forall f((\blacksquare ?Sf\backslash[]^{-1}[]^{-1}Sf)/\blacksquare Sf) : (\Phi^{n+}\ o\ \mathit{and}), \\
\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^{\wedge}\lambda I\lambda J(({}^{\vee}\mathit{eat}\ I)\ J), \\
\blacksquare\forall f\forall a((((\langle\rangle Na\backslash Sf){\uparrow}\blacksquare Nt(s(n))){\downarrow}(\blacksquare(\langle\rangle Na\backslash Sf)|\blacksquare Nt(s(n)))) : \lambda KK]]\ \Rightarrow\ Sf
\end{array}$$

$$\begin{array}{l}
[\blacksquare Nt(s(m)) : j], \square(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\square(\langle\rangle\exists gNt(s(g))\backslash Si)) : \\
{}^{\wedge}\lambda A\lambda B(({}^{\vee}\mathit{try}\ {}^{\wedge}({}^{\vee}A\ B))\ B), \blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash Si)/(\langle\rangle Nn\backslash Sb))) : \\
\lambda CC, [[\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^{\wedge}\lambda D\lambda E(({}^{\vee}\mathit{catch}\ D)\ E), \\
\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda F\lambda G\exists H[(F\ H)\wedge(G\ H)], \\
\square CNs(n) : \mathit{fish}, \blacksquare\forall f((\blacksquare ?Sf\backslash[]^{-1}[]^{-1}Sf)/\blacksquare Sf) : (\Phi^{n+}\ o\ \mathit{and}), \\
\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^{\wedge}\lambda I\lambda J(({}^{\vee}\mathit{eat}\ I)\ J), \\
\blacksquare[]^{-1}\forall f((\blacksquare Sf|\blacksquare Nt(s(n)))/(\langle\rangle Nt(s(n))\backslash Sf)) : \lambda KK]]\ \Rightarrow\ Sf
\end{array}$$

$$\begin{array}{l}
[\blacksquare Nt(s(m)) : j], \square(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\square(\langle\rangle\exists gNt(s(g))\backslash Si)) : \\
{}^{\wedge}\lambda A\lambda B(({}^{\vee}\mathit{try}\ {}^{\wedge}({}^{\vee}A\ B))\ B), \blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash Si)/(\langle\rangle Nn\backslash Sb))) : \\
\lambda CC, [[\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^{\wedge}\lambda D\lambda E(({}^{\vee}\mathit{catch}\ D)\ E), \\
\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda F\lambda G\exists H[(F\ H)\wedge(G\ H)], \\
\square CNs(n)\ : \mathit{fish}, \blacksquare\forall a\forall f((\blacksquare ?(\langle\rangle Na\backslash Sf)\backslash[]^{-1}[]^{-1}(\langle\rangle Na\backslash Sf))/\blacksquare(\langle\rangle Na\backslash Sf)) : \\
(\Phi^{n+}\ (s\ o)\ \mathit{and}), \square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^{\wedge}\lambda I\lambda J(({}^{\vee}\mathit{eat}\ I)\ J), \\
\blacksquare\forall f\forall a((((\langle\rangle Na\backslash Sf){\uparrow}\blacksquare Nt(s(n))){\downarrow}(\blacksquare(\langle\rangle Na\backslash Sf)|\blacksquare Nt(s(n)))) : \lambda KK]]\ \Rightarrow\ Sf
\end{array}$$

(Derivation omitted)

$$\exists C[({}^{\vee}\mathit{fish}\ C)\wedge(({}^{\vee}\mathit{try}\ {}^{\wedge}[(({}^{\vee}\mathit{catch}\ C)\ j)\wedge(({}^{\vee}\mathit{eat}\ C)\ j)])\ j)]$$

(Derivation omitted)

$$(({}^{\vee}\mathit{try}\ {}^{\wedge}\exists F[({}^{\vee}\mathit{fish}\ F)\wedge[(({}^{\vee}\mathit{catch}\ F)\ j)\wedge(({}^{\vee}\mathit{eat}\ F)\ j)]])\ j)$$

$$(({}^{\vee}\mathit{try}\ {}^{\wedge}\exists H[({}^{\vee}\mathit{fish}\ H)\wedge[(({}^{\vee}\mathit{catch}\ H)\ j)\wedge(({}^{\vee}\mathit{eat}\ H)\ j)]])\ j)$$

$$\begin{array}{l}
[\blacksquare Nt(s(m)) : j], \square(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\square(\langle\rangle\exists gNt(s(g))\backslash Si)) : \\
{}^{\wedge}\lambda A\lambda B(({}^{\vee}\mathit{try}\ {}^{\wedge}({}^{\vee}A\ B))\ B), \blacksquare((PPto/\exists aNa)\sqcap\forall n((\langle\rangle Nn\backslash Si)/(\langle\rangle Nn\backslash Sb))) : \\
\lambda CC, [[\square((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : {}^{\wedge}\lambda D\lambda E(({}^{\vee}\mathit{catch}\ D)\ E), \\
\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda F\lambda G\exists H[(F\ H)\wedge(G\ H)], \\
\square CNs(n)\ : \mathit{fish}, \blacksquare\forall a\forall f((\blacksquare ?(\langle\rangle Na\backslash Sf)\backslash[]^{-1}[]^{-1}(\langle\rangle Na\backslash Sf))/\blacksquare(\langle\rangle Na\backslash Sf)) :
\end{array}$$

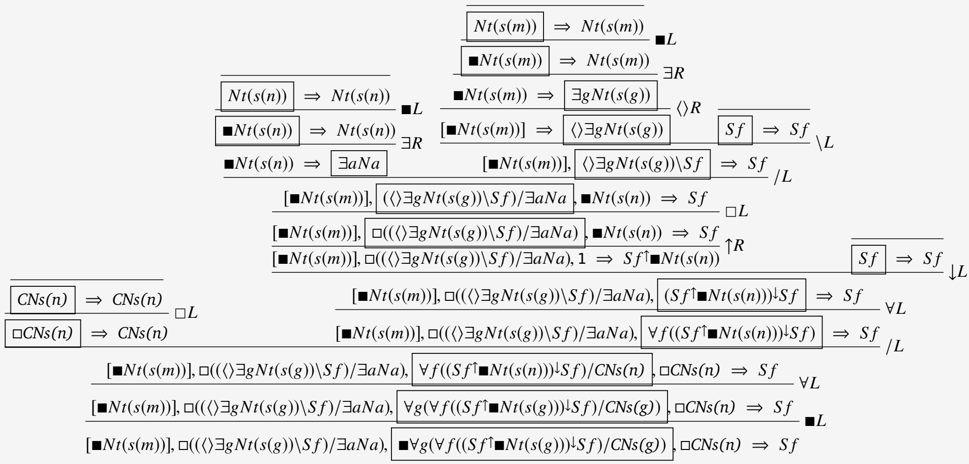

Figure 9 Derivation of (dwp((7-98)))

$(\Phi^{n+}\ (s\ 0)\ and), \Box(((\langle\rangle\exists aNa\backslash Sb)/\exists aNa) : \hat{}\lambda I\lambda J((\check{}eat\ I)\ J),$
$\blacksquare[]^{-1}\forall f((\blacksquare Sf|\blacksquare Nt(s(n)))/(\langle\rangle Nt(s(n))\backslash Sf)) : \lambda KK]] \Rightarrow Sf$

(dwp((7-98))) [**john**]+**finds**+**a**+**unicorn** : Sf

$[\blacksquare Nt(s(m)) : j], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \hat{}\lambda A\lambda B((\check{}find\ A)\ B),$
$\blacksquare\forall g(\forall f((Sf\uparrow\blacksquare Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda C\lambda D\exists E[(C\ E) \wedge (D\ E)],$
$\Box CNs(n) : unicorn \Rightarrow Sf$

For the derivation, see figure 9.

$\exists C[(\check{}unicorn\ C) \wedge ((\check{}find\ C)\ j)]$

(dwp((7-105))) [**every**+**man**+**such**+**that**+[**he**]+**loves**+**a**+**woman**] +**loses**+**her** : Sf

$[\blacksquare\forall g(\forall f((Sf\uparrow Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\Box CNs(m) : man, \blacksquare\forall n((CNn\backslash CNn)/(Sf|\blacksquare Nt(n))) : \lambda D\lambda E\lambda F[(E\ F) \wedge (D\ F)],$
$[\blacksquare[]^{-1}\forall g((\blacksquare Sg|\blacksquare Nt(s(m)))/(\langle\rangle Nt(s(m))\backslash Sg)) : \lambda GG],$
$\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \hat{}\lambda H\lambda I((\check{}love\ H)\ I),$
$\blacksquare\forall g(\forall f((Sf\uparrow\blacksquare Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda J\lambda K\exists L[(J\ L) \wedge (K\ L)],$
$\Box CNs(f) : woman],$
$\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \hat{}\lambda M\lambda N((\check{}lose\ M)\ N),$
$\blacksquare\forall g\forall a(((\langle\rangle Na\backslash Sg)\uparrow\blacksquare Nt(s(f)))\downarrow(\blacksquare(\langle\rangle Na\backslash Sg)|\blacksquare Nt(s(f)))) : \lambda OO \Rightarrow Sf$

(Derivation omitted)

$\exists C[(\check{}woman\ C) \wedge \forall G[[(\check{}man\ G) \wedge ((\check{}love\ C)\ G)] \rightarrow ((\check{}lose\ C)\ G)]]$

(dwp((7-110))) [**john**]+**walks**+**in**+**a**+**park** : Sf

$[\blacksquare Nt(s(m)) : j], \square(\langle\rangle\exists g Nt(s(g))\backslash Sf) : \hat{}\lambda A(\check{}walk\ A),$
$\square(\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))/\exists a Na) : \hat{}\lambda B\lambda C\lambda D((\check{}in\ B)\ (C\ D)),$
$\blacksquare\forall g(\forall f((Sf{\uparrow}\blacksquare Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda E\lambda F\exists G[(E\ G) \wedge (F\ G)],$
$\square CNs(n) : park \ \Rightarrow\ Sf$

(Derivation omitted)

$\exists C[(\check{}park\ C) \wedge ((\check{}in\ C)\ (\check{}walk\ j))]$

(dwp((7-116, 118))) [**every**+**man**]+**doesnt**+**walk** : Sf

$[\blacksquare\forall g(\forall f((Sf{\uparrow}Nt(s(g))){\downarrow}Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
$\square CNs(m) : man], \blacksquare\forall g\forall a((Sg{\uparrow}((\langle\rangle Na\backslash Sf)/(\langle\rangle Na\backslash Sb))){\downarrow}Sg) :$
$\lambda D\neg(D\ \lambda E\lambda F(E\ F)), \square(\langle\rangle\exists a Na\backslash Sb) : \hat{}\lambda G(\check{}walk\ G) \ \Rightarrow\ Sf$

(Derivation omitted)

$\forall C[(\check{}man\ C) \rightarrow \neg(\check{}walk\ C)]$

(Derivation omitted)

$\neg\forall G[(\check{}man\ G) \rightarrow (\check{}walk\ G)]$

Appendix: Rules

The syntactic types of displacement logic are sorted $\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2, \ldots$ according to the number of points of discontinuity $0, 1, 2, \ldots$ their expressions contain. Each type predicate letter has a sort and an arity which are naturals, and a corresponding semantic type. Assuming ordinary terms to be already given, where P is a type predicate letter of sort i and arity n and $t_1, \ldots, t_n$ are terms, $Pt_1 \ldots t_n$ is an (atomic) type of sort i of the corresponding semantic type. Compound types are formed by connectives as indicated in table 2,[2] and the structure preserving semantic type map T

[2]We list only connectives drawn from the first two rows of table 1, omitting some which are not central here.

1.	$\mathscr{F}_i$	::=	$\mathscr{F}_{i+j}/\mathscr{F}_j$	$T(C/B)$	=	$T(B) \to T(C)$	over
2.	$\mathscr{F}_j$	::=	$\mathscr{F}_i \backslash \mathscr{F}_{i+j}$	$T(A \backslash C)$	=	$T(A) \to T(C)$	under
3.	$\mathscr{F}_{i+j}$	::=	$\mathscr{F}_i \bullet \mathscr{F}_j$	$T(A \bullet B)$	=	$T(A) \& T(B)$	continuous product
4.	$\mathscr{F}_0$	::=	I	$T(I)$	=	$\top$	continuous unit
5.	$\mathscr{F}_{i+1}$	::=	$\mathscr{F}_{i+j} \uparrow_k \mathscr{F}_j, 1 \leq k \leq i+j$	$T(C \uparrow_k B)$	=	$T(B) \to T(C)$	extract
6.	$\mathscr{F}_j$	::=	$\mathscr{F}_{i+1} \downarrow_k \mathscr{F}_{i+j}, 1 \leq k \leq i+1$	$T(A \downarrow_k C)$	=	$T(A) \to T(C)$	infix
7.	$\mathscr{F}_{i+j}$	::=	$\mathscr{F}_{i+1} \odot_k \mathscr{F}_j, 1 \leq k \leq i+1$	$T(A \odot_k B)$	=	$T(A) \& T(B)$	discontinuous product
8.	$\mathscr{F}_1$	::=	J	$T(J)$	=	$\top$	discontinuous unit
9.	$\mathscr{F}_i$	::=	$\mathscr{F}_i \& \mathscr{F}_i$	$T(A \& B)$	=	$T(A) \& T(B)$	additive conjunction
10.	$\mathscr{F}_i$	::=	$\mathscr{F}_i \oplus \mathscr{F}_i$	$T(A \oplus B)$	=	$T(A) + T(B)$	additive disjunction
11.	$\mathscr{F}_i$	::=	$\bigwedge V \mathscr{F}_i$	$T(\bigwedge vA)$	=	$F \to T(A)$	1st order univ. qu.
12.	$\mathscr{F}_i$	::=	$\bigvee V \mathscr{F}_i$	$T(\bigvee vA)$	=	$F \& T(A)$	1st order exist. qu.
13.	$\mathscr{F}_i$	::=	$\Box \mathscr{F}_i$	$T(\Box A)$	=	$\mathbf{L}T(A)$	universal modality
14.	$\mathscr{F}_i$	::=	$\Diamond \mathscr{F}_i$	$T(\Diamond A)$	=	$\mathbf{M}T(A)$	existential modality
15.	$\mathscr{F}_i$	::=	$[\,]^{-1} \mathscr{F}_i$	$T([\,]^{-1} A)$	=	$T(A)$	univ. bracket modality
16.	$\mathscr{F}_i$	::=	$\langle\rangle \mathscr{F}_i$	$T(\langle\rangle A)$	=	$T(A)$	exist. bracket modality
17.	$\mathscr{F}_0$	::=	$!\mathscr{F}_0$	$T(!A)$	=	$T(A)$	universal exponential
18.	$\mathscr{F}_0$	::=	$?\mathscr{F}_0$	$T(?A)$	=	$T(A)^+$	existential exponential
19.	$\mathscr{F}_{i+j}$	::=	$\mathscr{F}_{i+j} \| \mathscr{F}_j$	$T(B \| A)$	=	$T(A) \to T(B)$	contr. for anaph.
35.	$\mathscr{F}_i$	::=	$\forall V \mathscr{F}_i$	$T(\forall vA)$	=	$T(A)$	sem. inactive 1st order univ. qu.
36.	$\mathscr{F}_i$	::=	$\exists V \mathscr{F}_i$	$T(\exists vA)$	=	$T(A)$	sem. inactive 1st order exist. qu.
37.	$\mathscr{F}_i$	::=	$\blacksquare \mathscr{F}_i$	$T(\blacksquare A)$	=	$T(A)$	sem. inactive universal modality
38.	$\mathscr{F}_i$	::=	$\blacklozenge \mathscr{F}_i$	$T(\blacklozenge A)$	=	$T(A)$	sem. inactive existential modality

Table 2 Syntactic types

associates these with semantic types.

In Gentzen sequent configurations (Γ, Δ) for displacement calculus a discontinuous type is a mother, rather than a leaf, and dominates its discontinuous components marked off by curly brackets and colons.

In Gentzen sequent antecedents for displacement logic with bracket modalities (structural inhibition) and exponentials (structural facilitation) there is also a bracket constructor for the former and 'stoups' for the latter.

Stoups (cf. the linear logic of Girard 2011 (ζ) are stores read as multisets for re-usable (nonlinear) resources which appear at the left of a configuration marked off by a semicolon (when the stoup is empty the semicolon may be omitted, as in the derivations of the previous section). The stoup of linear logic is for resources which can be contracted (copied) or weakened (deleted). By contrast, our stoup is for a linguistically motivated variant of contraction, and does not allow weakening. Furthermore, whereas linear logic is commutative, our logic is in general noncommutative and the stoup is used for resources which are also commutative.

A configuration together with a stoup is a *zone* (Ξ). The bracket constructor applies not to a configuration alone but to a configuration with a stoup, i.e a zone: reusable resources are specific to their domain.

Stoups $\mathcal{S}$ and configurations $\mathcal{O}$ are defined by ($\emptyset$ is the empty stoup; Λ is the empty configuration; the *separator* 1 marks points of discontinuity.:[3]

$$\text{(1)}\quad \begin{array}{lcl} \mathcal{S} & ::= & \emptyset \mid \mathcal{F}_0, \mathcal{S} \\ \mathcal{O} & ::= & \Lambda \mid \mathcal{T}, \mathcal{O} \\ \mathcal{T} & ::= & 1 \mid \mathcal{F}_0 \mid \mathcal{F}_{i>0}\{\underbrace{\mathcal{O} : \ldots : \mathcal{O}}_{i\ \mathcal{O}'\text{s}}\} \mid [\mathcal{S}; \mathcal{O}] \end{array}$$

For a type A, its sort $s(A)$ is the i such that $A \in \mathcal{F}_i$. For a configuration Γ, its sort $s(\Gamma)$ is $|\Gamma|_1$, that is, the number of points of discontinuity 1 which it contains. Sequents are of the form:

$$\text{(2)}\quad \mathcal{S}; \mathcal{O} \Rightarrow \mathcal{F} \text{ such that } s(\mathcal{O}) = s(\mathcal{F})$$

The figure $\overrightarrow{A}$ of a type A is defined by:

$$\text{(3)}\quad \overrightarrow{A} = \begin{cases} A & \text{if } s(A) = 0 \\ A\{\underbrace{1 : \ldots : 1}_{s(A)\ 1'\text{s}}\} & \text{if } s(A) > 0 \end{cases}$$

Where Γ is a configuration of sort i and $\Delta_1, \ldots, \Delta_i$ are configurations, the *fold* $\Gamma \otimes \langle \Delta_1 : \ldots : \Delta_i \rangle$ is the result of replacing the successive 1's in Γ by $\Delta_1, \ldots, \Delta_i$ respectively. Where Γ is of sort i, the hyperoccurrence notation $\Delta\langle\Gamma\rangle$ abbreviates $\Delta_0(\Gamma \otimes \langle \Delta_1 : \ldots : \Delta_i \rangle)$, that is, a context configuration Δ (which is externally Δ_0 and internally $\Delta_1, \ldots, \Delta_i$) with a potentially discontinuous distinguished subconfiguration Γ. Where Δ is a configuration of sort $i > 0$ and Γ is a configuration, the kth *metalinguistic intercalation* $\Delta \,|_k\, \Gamma$, $1 \leq k \leq i$, is given by:

$$\text{(4)}\quad \Delta \,|_k\, \Gamma =_{df} \Delta \otimes \langle \underbrace{1 : \ldots : 1}_{k-1\ 1\text{'s}} : \Gamma : \underbrace{1 : \ldots : 1}_{i-k\ 1\text{'s}} \rangle$$

that is, $\Delta \,|_k\, \Gamma$ is the configuration resulting from replacing by Γ the kth separator in Δ.

[3]Note that only types of sort 0 can go into the stoup; reusable types of other sorts would not preserve the sequent antecedent-succedent sort equality under contraction.

1. $$\frac{\zeta_1;\Gamma \Rightarrow B\colon \psi \qquad \zeta_2;\Delta\langle\overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\zeta_1 \uplus \zeta_2;\Delta\langle\overrightarrow{C/B}\colon x,\Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\}}/L \qquad \frac{\zeta;\Gamma,\overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\zeta;\Gamma \Rightarrow C/B\colon \lambda y\chi}/R$$

2. $$\frac{\zeta_1;\Gamma \Rightarrow A\colon \phi \qquad \zeta_2;\Delta\langle\overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\zeta_1 \uplus \zeta_2;\Delta\langle\Gamma,\overrightarrow{A\backslash C}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\}}\backslash L \qquad \frac{\zeta;\overrightarrow{A}\colon x,\Gamma \Rightarrow C\colon \chi}{\zeta;\Gamma \Rightarrow A\backslash C\colon \lambda x\chi}\backslash R$$

3. $$\frac{\zeta;\Delta\langle\overrightarrow{A}\colon x,\overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\zeta;\Delta\langle\overrightarrow{A{\bullet}B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x,\pi_2 z/y\}}\bullet L \qquad \frac{\zeta_1;\Gamma_1 \Rightarrow A\colon \phi \qquad \zeta_2;\Gamma_2 \Rightarrow B\colon \psi}{\zeta_1 \uplus \zeta_2;\Gamma_1,\Gamma_2 \Rightarrow A{\bullet}B\colon (\phi,\psi)}\bullet R$$

4. $$\frac{\zeta;\Delta\langle\Lambda\rangle \Rightarrow A\colon \phi}{\zeta;\Delta\langle\overrightarrow{I}\colon x\rangle \Rightarrow A\colon \phi}IL \qquad \frac{}{\emptyset;\Lambda \Rightarrow I\colon 0}IR$$

Figure 10 Continuous multiplicatives

A semantically labelled sequent is a sequent in which the antecedent type occurrences $A_1,\ldots,A_n$ are labelled by distinct variables $x_1,\ldots,x_n$ of types $T(A_1),\ldots,T(A_n)$ respectively, and the succedent type A is labelled by a term of type $T(A)$ with free variables drawn from $x_1,\ldots,x_n$. In this appendix we give the semantically labelled Gentzen sequent rules for some primary connectives, and indicate some linguistic applications.

The continuous multiplicatives of figure 10, the Lambek connectives (Lambek 1958, 1988), defined in relation to appending, are the basic means of categorial categorization and subcategorization. Note that here and throughout the active types in antecedents are figures (vectorial) whereas those in succedents are not; intuitively this is because antecedents are structured but succedents are not. The directional divisions over, /, and under, \, are exemplified by assignments such as **the**: N/CN for **the man**: N, **sings**: $N\backslash S$ for **John sings**: S, and **loves**: $(N\backslash S)/N$ for **John loves Mary**: S. The continuous product • is exemplified by a 'small clause' assignment such as **considers**: $(N\backslash S)/(N{\bullet}(CN/CN))$.

The discontinuous multiplicatives of figure 11, the displacement connectives (Morrill & Valentín 2010, Morrill et al. 2011), are defined in relation to plugging. When the value of the k subindex indicates the first (leftmost) point of discontinuity, it may be omitted. Extraction, ↑, is exemplified by a discontinuous idiom assignment **gives**+1+**the**+**cold**+**shoulder**: $(N\backslash S)\uparrow N$ for **Mary gives John the cold shoulder**: S, and infixation, ↓, and extrac-

5. $$\frac{\zeta_1; \Gamma \Rightarrow B\colon \psi \qquad \zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\zeta_1 \uplus \zeta_2; \Delta\langle \overrightarrow{C{\uparrow_k}B}\colon x \,|_k\, \Gamma\rangle \Rightarrow D\colon \omega\{(x\ \psi)/z\}}\,{\uparrow_k}L \qquad \frac{\zeta; \Gamma \,|_k\, \overrightarrow{B}\colon y \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow C{\uparrow_k}B\colon \lambda y \chi}\,{\uparrow_k}R$$

6. $$\frac{\zeta_1; \Gamma \Rightarrow A\colon \phi \qquad \zeta_2; \Delta\langle \overrightarrow{C}\colon z\rangle \Rightarrow D\colon \omega}{\zeta_1 \uplus \zeta_2; \Delta\langle \Gamma \,|_k\, \overrightarrow{A{\downarrow_k}C}\colon y\rangle \Rightarrow D\colon \omega\{(y\ \phi)/z\}}\,{\downarrow_k}L \qquad \frac{\zeta; \overrightarrow{A}\colon x \,|_k\, \Gamma \Rightarrow C\colon \chi}{\zeta; \Gamma \Rightarrow A{\downarrow_k}C\colon \lambda x \chi}\,{\downarrow_k}R$$

7. $$\frac{\zeta; \Delta\langle \overrightarrow{A}\colon x \,|_k\, \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\zeta; \Delta\langle \overrightarrow{A{\odot_k}B}\colon z\rangle \Rightarrow D\colon \omega\{\pi_1 z/x, \pi_2 z/y\}}\,{\odot_k}L \qquad \frac{\zeta_1; \Gamma_1 \Rightarrow A\colon \phi \qquad \zeta_2; \Gamma_2 \Rightarrow B\colon \psi}{\zeta_1 \uplus \zeta_2; \Gamma_1 \,|_k\, \Gamma_2 \Rightarrow A{\odot_k}B\colon (\phi, \psi)}\,{\odot_k}R$$

8. $$\frac{\zeta; \Delta\langle 1\rangle \Rightarrow A\colon \phi}{\zeta; \Delta\langle \overrightarrow{J}\colon x\rangle \Rightarrow A\colon \phi}\,JL \qquad \frac{}{\emptyset; 1 \Rightarrow J\colon 0}\,JR$$

Figure 11 Discontinuous multiplicatives

9. $$\frac{\Xi\langle \overrightarrow{A}\colon x\rangle \Rightarrow C\colon \chi}{\Xi\langle \overrightarrow{A\&B}\colon z\rangle \Rightarrow C\colon \chi\{\pi_1 z/x\}}\,\&L_1 \qquad \frac{\Xi\langle \overrightarrow{B}\colon y\rangle \Rightarrow C\colon \chi}{\Xi\langle \overrightarrow{A\&B}\colon z\rangle \Rightarrow C\colon \chi\{\pi_2 z/y\}}\,\&L_2 \qquad \frac{\Xi \Rightarrow A\colon \phi \qquad \Xi \Rightarrow B\colon \psi}{\Xi \Rightarrow A\&B\colon (\phi, \psi)}\,\&R$$

10. $$\frac{\Xi\langle \overrightarrow{A}\colon x\rangle \Rightarrow C\colon \chi_1 \qquad \Xi\langle \overrightarrow{B}\colon y\rangle \Rightarrow C\colon \chi_2}{\Xi\langle \overrightarrow{A\oplus B}\colon z\rangle \Rightarrow C\colon z \to x.\chi_1; y.\chi_2}\,\oplus L \qquad \frac{\Xi \Rightarrow A\colon \phi}{\Xi \Rightarrow A\oplus B\colon \iota_1\phi}\,\oplus R_1 \qquad \frac{\Xi \Rightarrow B\colon \psi}{\Xi \Rightarrow A\oplus B\colon \iota_2\psi}\,\oplus R_2$$

Figure 12 Additives

tion together are exemplified by a quantifier phrase assignment **everyone**: $(S{\uparrow}N){\downarrow}S$, simulating Montague's S14 treatment of quantifying in. Extraction and discontinuous product, $\odot$, are shown together with the continuous unit in an assignment to a relative pronoun **that**: $(CN\backslash CN)/((S{\uparrow}N){\odot}I)$, allowing both peripheral and medial extraction, as in **that John likes**: $CN\backslash CN$ and **that John saw today**: $CN\backslash CN$.

In relation to the multiplicative rules, notice how the stoup is distributed reading bottom-up from conclusions to premise: it is partitioned between the two premises in the case of binary rules, copied to the premise in the case of unary rules, and empty in the case of nullary rules (axioms).

The remaining figures give rules for additives, quantifiers, normal modalities, bracket modalities, exponentials, and limited contraction for anaphora.

Acknowledgments This research was partially financed by an ICREA Acadèmia 2012 to Morrill, MINECO project APCOM (TIN2014-57226-P), and Generalitat de Catalunya grant SGR2014-890 (MACDA). Many thanks to C. Piñón for

11. $$\frac{\Xi\langle\overrightarrow{A[t/v]\colon x}\rangle \Rightarrow B\colon\psi}{\Xi\langle\overrightarrow{\bigwedge vA\colon z}\rangle \Rightarrow B\colon\psi\{(z\ t)/x\}}\wedge L \qquad \frac{\Xi \Rightarrow A[a/v]\colon\phi}{\Xi \Rightarrow \bigwedge vA\colon \lambda v\phi}\wedge R^{\dagger}$$

12. $$\frac{\Xi\langle\overrightarrow{A[a/v]\colon x}\rangle \Rightarrow B\colon\psi}{\Xi\langle\overrightarrow{\bigvee vA\colon z}\rangle \Rightarrow B\colon\psi\{\pi_2 z/x\}}\vee L^{\dagger} \qquad \frac{\Xi \Rightarrow A[t/v]\colon\phi}{\Xi \Rightarrow \bigvee vA\colon (t,\phi)}\vee R$$

Figure 13 Quantifiers, where † indicates that there is no a in the conclusion

13. $$\frac{\Xi\langle\overrightarrow{A\colon x}\rangle \Rightarrow B\colon\psi}{\Xi\langle\overrightarrow{\Box A\colon z}\rangle \Rightarrow B\colon\psi\{^{\vee}z/x\}}\Box L \qquad \frac{\boxtimes\Xi \Rightarrow A\colon\phi}{\boxtimes\Xi \Rightarrow \Box A\colon {}^{\wedge}\phi}\Box R$$

14. $$\frac{\boxtimes\Xi\langle\overrightarrow{A\colon x}\rangle \Rightarrow \diamondplus B\colon\psi}{\boxtimes\Xi\langle\overrightarrow{\Diamond A\colon z}\rangle \Rightarrow \diamondplus B\colon\psi\{^{\cup}z/x\}}\Diamond L \qquad \frac{\Xi \Rightarrow A\colon\phi}{\Xi \Rightarrow \Diamond A\colon {}^{\cap}\phi}\Diamond R$$

Figure 14 Normal modalities, where $\boxtimes$/$\diamondplus$ marks a structure all the types of which have main connective a box/diamond

15. $$\frac{\Xi\langle\overrightarrow{A\colon x}\rangle \Rightarrow B\colon\psi}{\Xi\langle\overrightarrow{[[\,]^{-1}A\colon x]}\rangle \Rightarrow B\colon\psi}[\,]^{-1}L \qquad \frac{[\Xi] \Rightarrow A\colon\phi}{\Xi \Rightarrow [\,]^{-1}A\colon\phi}[\,]^{-1}R$$

16. $$\frac{\Xi\langle[\overrightarrow{A\colon x}]\rangle \Rightarrow B\colon\psi}{\Xi\langle\overrightarrow{\langle\rangle A\colon x}\rangle \Rightarrow B\colon\psi}\langle\rangle L \qquad \frac{\Xi \Rightarrow A\colon\phi}{[\Xi] \Rightarrow \langle\rangle A\colon\phi}\langle\rangle R$$

Figure 15 Bracket modalities

17. $$\frac{\Xi(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2) \Rightarrow B\colon\psi}{\Xi(\zeta; \Gamma_1, !A\colon x, \Gamma_2) \Rightarrow B\colon\psi}!L \qquad \frac{\zeta;\Lambda \Rightarrow A\colon\phi}{\zeta;\Lambda \Rightarrow !A\colon\phi}!R \qquad \frac{\Xi(\zeta; \Gamma_1, A\colon x, \Gamma_2) \Rightarrow B\colon\psi}{\Xi(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2) \Rightarrow B\colon\psi}!P \qquad \frac{\Xi(\zeta \uplus \{A\colon x\}; \Gamma_1, [\{A\colon y\}; \Gamma_2], \Gamma_3) \Rightarrow B\colon\psi}{\Xi(\zeta \uplus \{A\colon x\}; \Gamma_1, \Gamma_2, \Gamma_3) \Rightarrow B\colon\psi\{x/y\}}!C$$

18. $$\frac{\Delta(A\colon x) \Rightarrow D\colon\omega([x]) \quad \Delta(A\colon x, A\colon y) \Rightarrow D\colon\omega([x,y]) \quad \ldots}{\Delta(?A\colon w) \Rightarrow D\colon\omega(w)}?L \qquad \frac{\Xi \Rightarrow A\colon\phi}{\Xi \Rightarrow ?A\colon[\phi]}?R \qquad \frac{\zeta;\Gamma \Rightarrow A\colon\phi \quad \zeta';\Delta \Rightarrow ?A\colon\psi}{\zeta \uplus \zeta';\Gamma,\Delta \Rightarrow ?A\colon[\phi|\psi]}?M$$

Figure 16 Exponentials

19. $$\frac{\zeta; \Gamma \Rightarrow A\colon \phi \qquad \zeta'; \Delta\langle \overrightarrow{A}\colon x; \overrightarrow{B}\colon y\rangle \Rightarrow D\colon \omega}{\zeta \uplus \zeta'; \Delta\langle \Gamma; \overrightarrow{B|A}\colon z\rangle \Rightarrow D\colon \omega\{\phi/x, (z\ \phi)/y\}}|L \qquad \frac{\zeta; \Gamma\langle \overrightarrow{B_0}\colon y_0; \ldots; \overrightarrow{B_n}\colon y_n\rangle \Rightarrow D\colon \omega}{\zeta; \Gamma\langle \overrightarrow{B_0|A}\colon z_0; \ldots; \overrightarrow{B_n|A}\colon z_n\rangle \Rightarrow D|A\colon \lambda x \omega\{(z_0\ x)/y_0, \ldots, (z_n\ x)/y_n\}}|R$$

Figure 17 Limited contraction for anaphora

valuable editorial guidance, and for anonymous EISS reviewing. All errors are our own.

References

Andreoli, J. M. 1992. Logic programming with focusing proofs in linear logic. *Journal of Logic and Computation* 2(3). 297–347. doi:10.1093/logcom/2.3.297.

Carpenter, Bob. 1997. *Type-logical semantics*. Cambridge, MA: The MIT Press.

Dowty, David R., Robert E. Wall & Stanley Peters. 1981. *Introduction to Montague semantics*, vol. 11 Synthese Language Library. Dordrecht: D. Reidel.

Girard, Jean-Yves. 2011. *The blind spot*. Zürich: European Mathematical Society.

Hepple, Mark. 1990. *The grammar and processing of order and dependency*: University of Edinburgh dissertation.

Jäger, Gerhard. 2005. *Anaphora and type logical grammar*, vol. 24 Trends in Logic – Studia Logica Library. Springer.

Lambek, Joachim. 1958. The mathematics of sentence structure. *American Mathematical Monthly* 65. 154–170.

Lambek, Joachim. 1988. Categorial and categorical grammars. In Richard T. Oehrle, Emmon Bach & Deidre Wheeler (eds.), *Categorial grammars and natural language structures*, vol. 32 Studies in Linguistics and Philosophy, 297–317. Dordrecht: D. Reidel.

Montague, Richard. 1970a. English as a formal language. In B. Visentini et al. (ed.), *Linguaggi nella società e nella tecnica*, 189–224. Milan: Edizioni di Comunità. Reprinted in R. H. Thomason, editor, 1974, *Formal Philosophy: Selected Papers of Richard Montague*, Yale University Press, New Haven, CT, 188–221.

Montague, Richard. 1970b. Universal grammar. *Theoria* 36. 373–398. Reprinted in R. H. Thomason, editor, 1974, *Formal Philosophy: Selected Papers of Richard Montague*, Yale University Press, New Haven, CT, 222–246.

Montague, Richard. 1973. The proper treatment of quantification in ordinary English. In J. Hintikka, J. M. E. Moravcsik & P. Suppes (eds.), *Approaches to natural language: Proceedings of the 1970 Stanford workshop on grammar and semantics*, 189–224. Dordrecht: D. Reidel. Reprinted in R. H. Thomason, editor, 1974, *Formal Philosophy: Selected Papers of Richard Montague*, Yale University Press, New Haven, CT, 247–270.

Moortgat, Michael. 1988. *Categorial investigations: Logical and linguistic aspects* *of the Lambek calculus*. Dordrecht: Foris.

Moortgat, Michael. 1996. Multimodal linguistic inference. *Journal of Logic, Language and Information* 5(3, 4). 349–385. Also in *Bulletin of the IGPL*, 3(2, 3): 371–401, 1995.

Moortgat, Michael. 1997. Categorial type logics. In Johan van Benthem & Alice ter Meulen (eds.), *Handbook of logic and language*, 93–177. Amsterdam and Cambridge, Massachusetts: Elsevier Science B.V. and the MIT Press.

Moot, Richard & Christian Retoré. 2012. *The logic of categorial grammars: A deductive account of natural language syntax and semantics*. Springer.

Morrill, Glyn. 1990a. Grammar and logical types. In Martin Stockhof & Leen Torenvliet (eds.), *Proceedings of the Seventh Amsterdam Colloquium*, 429–450. Universiteit van Amsterdam.

Morrill, Glyn. 1990b. Intensionality and boundedness. *Linguistics and Philosophy* 13(6). 699–726.

Morrill, Glyn. 1992. Categorial formalisation of relativisation: Pied piping, islands, and extraction sites. Tech. Rep. LSI-92-23-R Departament de Llenguatges i Sistemes Informàtics, Universitat Politècnica de Catalunya.

Morrill, Glyn & Josep-Maria Merenciano. 1996. Generalising discontinuity. *Traitement automatique des langues* 37(2). 119–143.

Morrill, Glyn & Oriol Valentín. 2010. Displacement calculus. *Linguistic Analysis* 36(1–4). 167–192. `http://arxiv.org/abs/1004.4181`. Special issue Festschrift for Joachim Lambek.

Morrill, Glyn & Oriol Valentín. 2014a. Displacement logic for anaphora. *Journal of Computing and System Science* 80(2). 390–409.

Morrill, Glyn & Oriol Valentín. 2014b. Semantically inactive multiplicatives and words as types. In Nicholas Asher & Sergei Soloviev (eds.), *Proceedings of Logical Aspects of Computational Linguistics, LACL'14, Toulouse* (LNCS, FoLLI Publications on Logic, Language and Information 8535), 149–162. Springer.

Morrill, Glyn & Oriol Valentín. 2015a. Computational coverage of TLG: Nonlinearity. In M. Kanazawa, L. S. Moss & V. de Paiva (eds.), *Proceedings of NLCS'15, Third Workshop on Natural Language and Computer Science*, vol. 32 EPiC, 51–63. Kyoto. Workshop affiliated with Automata, Languages and Programming (ICALP) and Logic in Computer Science (LICS).

Morrill, Glyn & Oriol Valentín. 2015b. Multiplicative-additive focusing for parsing as deduction. In I. Cervesato & C. Schürmann (eds.), *First International Workshop on Focusing, workshop affiliated with LPAR 2015* (EPTCS 197), 29–54. Suva, Fiji.

Morrill, Glyn, Oriol Valentín & Mario Fadda. 2009. Dutch grammar and processing: A case study in TLG. In Peter Bosch, David Gabelaia & Jérôme Lang (eds.), *Logic, language, and computation: 7th International Tbilisi Symposium, Revised selected papers* (Lecture Notes in Artificial Intelligence 5422), 272–286. Springer.

Morrill, Glyn, Oriol Valentín & Mario Fadda. 2011. The displacement calculus. *Journal of Logic, Language and Information* 20(1). 1–48.

Morrill, Glyn V. 1994. *Type logical grammar: Categorial logic of signs*. Kluwer.

Morrill, Glyn V. 2011. *Categorial grammar: Logical syntax, semantics, and processing*. Oxford University Press.