The Game of Same and Different — A Framework for Analyzing Determiner Systems

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Abstract

This paper proposes the complete *Game of Same and Different* as a framework for reasoning about determiner systems in natural languages. The system is applied to a Latin-like language with a singular-plural system, and no articles. It will be shown that the system converges under highly rational agents toward the use of the demonstrative as a definite article, and against the emergence of an indefinite article. The behavior of agents more limited in their rationality is explored, and compared with the diachronic development of definite and indefinite articles.

1 Introduction

One core idea of structuralism in linguistics is that linguistic items form *systems*, where the behavior and distribution of a single item is not only determined by its intrinsic meaning, but also by the meanings of other items it competes with (see Jakobson, 1932/1971: for an application to verbal categories). While the intuition itself has come back into favor in recent years (see, e.g. Sauerland, 2008), as far as I am aware, there have been no tentatives as complete as the one by Jakobson in order to account for a reasonably complex, and possibly full, linguistic subsystem. One of the reasons is the wealth of interacting constraints, which are difficult to reason through verbally with any degree of confidence. This paper proposes to overcome these difficulties by providing a framework for determiner systems in natural languages, using a Rational Speech Act (henceforth RSA) solution concept[1] (see, e.g., Franke, 2017).

[1] The pragmatic part of the game could be implemented with minor changes in an Iterated Quantal Response framework (see Franke and Jäger, 2014).

2 The Game of Same and Different

The *game of same and different* is a signalling game, with in the most general case n states of the world that a speaker signals, and for which m different messages can be used. The hearer then tries to map the sent message back to a state of the world. The game speaker and hearer are playing is a full *game of same and different* (for more limited versions, see Grønn and Szæbø, 2012; Amsili and Beyssade, 2016). In this game, the common ground may contain an element satisfying predicate P, and this entity could be singular or plural (or dual, or any other number available in the language). Then the speaker signals whether a newly introduced entity is the same as the one in the common ground, or not. The setup does not only contain identity and full difference (as considered previously in the literature), but also cases where the newly introduced entity is part of an entity already in the common ground, or where the newly introduced entity contains the entity in the common ground. I will presuppose in what follows a language with a singular and a plural, but other configurations could be considered, and the game itself could be accommodated to include further parameters (e.g., proximity of the given entity, salience, . . .).

2.1 States in the Game of Same and Different

In order to provide a formalization, we will first need to set up the different states of the world that a speaker might want to signal.

The basic meaning components can be stated as in (1) — concerning the entity in the common ground — and (2) — concerning the assertion — adopting Sauerland's flat DRT notation (where numerator = presupposed content; denominator = asserted content):

(1) a. $[\![\text{empty CG}]\!] = \frac{|}{\bullet}$

b. $[\![\text{non-empty CG}]\!] = \frac{\mathbb{x}|P(\mathbb{x})}{\bullet}$

(2) a. $[\![\text{existence}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}),Q(\mathbb{z})}$

b. $[\![\text{identity}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}),Q(\mathbb{z}),\mathbb{z}=\mathbb{x}}$

c. $[\![\text{partitive}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}),Q(\mathbb{z}),\mathbb{z}\sqsubset\mathbb{x}}$

d. $[\![\text{superset}]\!] = \frac{\bullet}{\mathbb{z}|P(\mathbb{z}),Q(\mathbb{z}),\mathbb{x}\sqsubset\mathbb{z}}$

In (1)–(2), $\mathbb{x}$ denotes a number-neutral variable, $\bullet$ stands for some DRT-formula, and $\sqsubset$ corresponds to either $\in$ or $\subset$. (1a) denotes an empty common ground (henceforth: CG) wrt P, whereas in (1b), some (singular or plural) entity satisfies P in the CG. (2a) illustrates a new entity wrt to the CG, (2b) an entity that is identical to the one in the CG, (2c) an entity that is a part of an entity in the CG, and (2d) an entity that is a superset wrt an entity in the CG.

Given this game, and the assumption that the language under consideration has a singular-plural system, these basic ingredients can be combined in 12 different ways in order to describe 12 differing states of the world. For instance, a state where the common ground is empty, and there is a simple assertion with respect to a singular entity (something that in English would call for an indefinite article) can be obtained by combining (1a) with (2b), while specifying the variable for a singular – which gives us (3a); a plural entity satisfying P in the CG, and asserting the identity of some entity with this (which in English would correspond to a definite plural) requires the combination of (1b) with (2b), and specifying the variable for plurals, which is illustrated in (3b):

(3) a. $\frac{|}{z|P(z),Q(z)}$

b. $\frac{X|P(X)}{Z|P(Z),Q(Z),Z=X}$

Not all combinations are possible: e.g., if the assertion is a partitive wrt the CG, the entity in the CG needs to be plural. Notice also that the superset condition could be derived compositionally, with an identity condition plus the explicit addition of new entities (see English, *"these and other Ns"*), and does not have to be stated as a primitive.

The states to be signaled (as used later in figure 1) are labeled as follows: the ‘E’, ‘S’ or ‘P’ in the beginning indicate that the CG is empty, singular or plural, respectively; the letter in the middle indicates whether the entity carrying the assertion are *N*ew, *I*dentical, *P*art or a *S*uperset wrt the entity in the CG; and the letter at the end indicates whether the number in the assertion is *S*ingular or *P*lural.

2.2 Forms in the Game of Same and Different

While the choice of states (that is, in the end, meanings) is arguably only dependent on grammatical number in a language, the choice of linguistic forms in order to express these meanings requires more assumptions. For the sake of the argument, I will suppose a language without grammaticalized determiner system, having a demonstrative determiner, an expression of type ‘*other*’, and a plural element of type ‘*several*’ that is to be taken into account (this is inspired by, though not completely identical to, a language such as Latin).

(4) a. $[\![\text{bare SG}]\!] = \frac{|}{z|P(z),Q(z)}$

b. $[\![\text{dem SG}]\!] = \frac{x|P(x)}{z|P(z),Q(z),z=x}$

c. $[\![\text{one SG}]\!] = \frac{|}{z|P(z),Q(z),|P\cap Q|=1}$

d. $[\![\text{other SG}]\!] = \frac{\mathbb{x}|P(\mathbb{x})}{z|P(z),Q(z),z\neq\mathbb{x}}$

e. $[\![\text{bare PL}]\!] = \frac{|}{Z|P(Z),Q(Z)}$

f. $[\![\text{dem PL}]\!] = \frac{X|P(X)}{Z|P(Z),Q(Z),Z=X}$

g. $[\![\text{sev PL}]\!] = \frac{|}{Z|P(Z),Q(Z),|P\cap Q|\geq n}$

h. $[\![\text{other PL}]\!] = \frac{\mathbb{x}|P(\mathbb{x})}{Z|P(Z),Q(Z),Z\neq\mathbb{x}}$

Given the forms in (4) at our disposal, and the states to signal, we can now establish a Boolean matrix, indicating for each form whether it is grammatical in a given state (and hence, marked by 1 in the matrix), or ungrammatical (and hence, marked by 0).

The black numbers in figure 1 indicate for each form whether it is grammatical in order to signal a given state, assuming that we are in a context where there is no interference from some quantifier.[2] This can be seen as a model of competence, and assumes that plurals are inclusive (and that the bare plural would be appropriate for absolutely anything).

A literal speaker (or Speaker_0), when confronted with a state to signal (assume that to be ENS), will simply check which forms are gram-

[2]Therefore, the matrix has to be seen as representing one particular context of use for determiners, and one should be aware that other contexts do exist (see Grønn and Szæbø, 2012).

	Bare SG	dem SG	1 SG	other SG	Bare PL	dem PL	sev PL	other PL
ENS	1|1	0|0	1|0	0|0	1|0	0|0	1|0	0|0
ENP	0|0	0|0	0|0	0|0	1|1	0|0	1|0	0|0
SNS	1|0	0|0	1|0	1|1	1|0	0|0	1|0	1|0
SNP	0|0	0|0	0|0	0|0	1|0	0|0	1|0	1|1
PNS	1|0	0|0	1|0	1|1	1|0	0|0	1|0	1|0
PNP	0|0	0|0	0|0	0|0	1|0	0|0	1|0	1|1
SIS	1|0	1|1	1|0	0|0	1|0	1|0	1|0	0|0
PIP	0|0	0|0	0|0	0|0	1|0	1|1	1|0	0|0
PPS	1|0	0|0	1|1	0|0	1|0	0|0	1|0	0|0
PPP	0|0	0|0	0|0	0|0	1|0	0|0	1|1	0|0
SSP	0|0	0|0	0|0	0|0	1|0	0|0	1|1	0|0
PSP	0|0	0|0	0|0	0|0	1|0	0|0	1|1	0|0

Figure 1: Boolean Matrix for Game of Same and Different. Black: competence; Red: speaker behavior predicted by RSA-model (for Speaker_{10} with λ=5)

matical (namely bare SG, 1 SG, bare PL, sev PL), and then choose uniformly at random one form (that is, with probability 0.25). A literal hearer (or Hearer_0), when confronted with a form (assume that to be bare SG), will check which states are compatible with that form (namely ENS, SNS, PNS, SIS, PPS) and then choose uniformly at random one state (that is, with probability 0.2).

This competence model is the input to a Rational Speech Act model as formalized for instance in Franke (2017). The idea is that a minimally pragmatic speaker (or Speaker_1) will choose the form to signal anticipating the reaction of a literal hearer. Similarly, a minimally pragmatic hearer (or Hearer_1) will anticipate a literal speaker in order to maximize the success of the communication. More generally, a Speaker_n will anticipate a Hearer_{n-1}, and similarly for a hearer. This can be formalized as in Franke (ibid.):

(5) a. $\text{Speaker}_{n+1} = \frac{\exp(\lambda * \text{EU}_S(m,t,\text{Hearer}_n))}{\sum_{m'} \exp(\lambda * \text{EU}_S(m',t,\text{Hearer}_n)))}$

b. $\text{Hearer}_{n+1} = \frac{Pr(t) \times \text{Speaker}_{n+1}(m|t)}{\sum_{t'} Pr(t') \times \text{Speaker}_{n+1}(m|t')}$

where m, m' = linguistic forms sent by the speaker, t, t' = states of the world, and λ is the softmax-parameter, indicating on how rational the agents will perform. The Expected Utility of a speaker, EU_S is defined as follows:

(6) $\text{EU}_S(m,t,\text{Hearer}_n) = \log(\text{Hearer}_n(t|m)) + \text{Hearer}_n(t|m) \times c_m$

If given sufficient leeway to converge (for instance, by choosing a highly sophisticated speaker of level 10, and a softmax-parameter of 5), this will produce the prediction of behavior as indicated in red in figure 1.

As can be seen, such a speaker will not use the plural at all in order to signal cases where the assertion is singular, even though it is systematically available in the grammar. Furthermore (and contrary to what happens in actually existing Latin), the system predicts that for definite uses (that is, cases SIS and PIP), the demonstrative should be systematically used. Notice that this is a simple scalar implicature, since the demonstrative is more specific/restraint than the bare form.

In addition, we obtain a manner-implicature between the bare singular and "one N", under the assumption that the bare form is less costly than the form determined by the unity cardinal, and further assuming that the state ENS is more frequent than state PPS. The same configuration also holds between the bare plural and "several Ns". Even though in principle, both forms would be possible in both contexts, by Horn's division of pragmatic labor, they specialize. This is arguably what happens, since in English, the same pattern is observable (although in the singular, the opposition is to the indefinite article, and not the bare singular):

(7) John showed me his books_i.

a. One $\text{book}_{j \in i}$ immediately caught my eye.

b. #A $\text{book}_{j \in i}$ immediately caught my eye.

c. Several $\text{books}_{j \in i}$ immediately caught my eye.

d. #$\text{Books}_{j \in i}$ immediately caught my eye.

This pattern is predicted as long as it is the case that i) *one N/several Ns* is more costly than the bare form; and ii) the state EPS (or EPP) is more frequent than PPS (or PPP).

3 Notes on Diachrony

The RSA-model I have presented predicts thus that — when played by sufficiently rational speakers, and who know perfectly the situation in which they are in — the demonstrative should completely take over states SIS and PIP (that is, uses indicating identity between CG and the newly asserted content). In other words, standard pragmatic procedures are sufficient to explain the emergence of a definite article, as soon as the demonstrative has become an alternative in the game. However, the situation is more complicated wrt indefinite articles: the model predicts a manner-implicature of the bare form vs. the unity cardinal (for the context of ENS for the bare SG, vs. partitive use, i.e., PPS, for ONE SG), which weighs in against the emergence of an indefinite article.

Yet, contrary to the prediction, in actual Latin, demonstratives where not (at least: not systematically) used in contexts of definite uses (that is, in states SIS and PIP); indeed the bare version was the most frequent case, as is illustrated in (8), from the Vulgate. Whatever is the exact reference of *Verbum* here, it is reasonably clear that the three occurrences of *Verbum* point to the same entity (and therefore, occurrences 2-3 illustrate state SIS)[3]. So, an explanation on the non-appearance of a demonstrative is required.

(8) in principio erat *Verbum* et *Verbum* erat apud Deum et Deus erat *Verbum*. [John 1:1; my emphasis]
In the beginning was the Word: and the Word was with God: and the Word was God.

A second divergence from reality comes from the assumption that speakers are always fully rational, and do not necessarily second-guess the hearers to a deep level of recursion; we are boundedly rational creatures, and do not always know precisely the state of the world we are confronted with. As reported in Franke (2017), psycholinguistic studies generally assume speakers of level 1 (which is *much* less sophisticated than what we have assumed above). Fully formalized models as the one above have the advantage of not only allowing for categorial prediction of speaker behavior, but also the investigation of limited rationality.

3.1 Demonstratives as Definite Articles

Why do we not systematically see demonstratives act as definite articles, as predicted by the model? The simplest answer to this is that the demonstrative does not enter into the list of alternatives, but why should this be so? Part of the answer probably lies in the fact that many languages (among which Latin, but also contemporary German or Spanish) do not simply have *one* demonstrative determiner, but rather several, which are differenciated along some proximity scale (cf., e.g. also German *dieses* – proximal vs. *jenes* – distal). Deléani and Vermandern (2003: 115ff.) point out that (classical) Latin had three demonstrative determiners, namely *hic, iste* and *ille* (the latter being the ancestor of most Romance definite articles).

As long as there is a functional differentiation along these lines, there simply is no one obvious candidate, which could compete with bare singulars or plurals for the SIS and PIP slots, respectively.

A similar reasoning can be adduced for the divergence of the frequency of the grammaticalization of indefinite articles for the singular (rather frequent) versus the plural (where they are much rarer). While there is one obvious candidate for an indefinite singular article (the unity cardinal *one*), there is no single obvious and salient candidate available for the plural (possible candidates would be equivalents of *several*, *a few*, the plural of *one*, ...).

3.2 The Impact of Limited Rationality

The RSA-model can be used in order to investigate the impact of limiting rationality, and doing it by using two *a priori* different measures of speaker sophistication or rationality, namely the softmax-parameter λ and the level of recursion in reasoning against ever more sophisticated hearers. While we assumed above a very sophisticated speaker with respect to the level of recursion, we will now investigate the effects of a much less sophisticated, and slightly less rational, speaker.

Consider the behavior of a Speaker$_1$ with $\lambda = 3$, as illustrated in figure 2. Here, we no longer have categorical predictions for the use of one form in one given state, but rather probabilities. For instance, if we look at the predicted behavior of such

[3] A similar argument could be made for the second occurrence of *God*.

	bare SG	dem SG	1 SG	other SG	bare PL	dem PL	sev PL	other PL
ENS	0.503490	0.000000	0.446555	0.000000	0.025602	0.000000	0.024353	0.000000
ENP	0.000000	0.000000	0.000000	0.000000	0.512497	0.000000	0.487503	0.000000
SNS	0.040410	0.000000	0.035841	0.852315	0.002055	0.000000	0.001955	0.067425
SNP	0.000000	0.000000	0.000000	0.000000	0.028765	0.000000	0.027362	0.943873
PNS	0.040410	0.000000	0.035841	0.852315	0.002055	0.000000	0.001955	0.067425
PNP	0.000000	0.000000	0.000000	0.000000	0.028765	0.000000	0.027362	0.943873
SIS	0.001272	0.961802	0.001128	0.000000	0.000065	0.035672	0.000062	0.000000
PIP	0.000000	0.000000	0.000000	0.000000	0.001807	0.996475	0.001718	0.000000
PPS	0.503490	0.000000	0.446555	0.000000	0.025602	0.000000	0.024353	0.000000
PPP	0.000000	0.000000	0.000000	0.000000	0.512497	0.000000	0.487503	0.000000
SSP	0.000000	0.000000	0.000000	0.000000	0.512497	0.000000	0.487503	0.000000
PSP	0.000000	0.000000	0.000000	0.000000	0.512497	0.000000	0.487503	0.000000

Figure 2: Limited Rationality: Predicted Behavior for Speaker_1 with $\lambda = 3$

a speaker for state SIS, we see that DEM SG still is by far the most frequent choice, but in rare cases, the bare versions of singular and plural will be used, and DEM PL will also appear in such environments. The case for state ENS is even more removed from what we have seen in the more rational and sophisticated behavior in figure 1: the unity cardinal *one* will be used in nearly 45% of cases.

One of the striking facts in this model is thus that standard pragmatic processes and rationality should act to advance the grammaticalisation of demonstrative determiners into definite articles, whereas the same processes should actively act against the grammaticalisation of the cardinal *one* into indefinite articles. Put otherwise: rationality will give us a definite article, but lack of rationality favours the indefinite. In order to show this, we can consider what kind of impact have the different parameters of recursion depth and rationality.

In figure 3, each line shows a given recursion depth for the speaker, along different values for λ, and the frequency of the bare noun is substracted from the frequency of the demonstrative in cases of singular identity. This shows that in case of demonstrative determiners, even in unsophisticated and not very rational speakers, the demonstrative is more frequent than the bare noun, and convergence towards all-demonstrative happens with very low recursion-depth, and low values for λ. However, in case of the unity cardinal *one*, the frequencies diverge much more slowly, and when they do diverge, they diverge in favor of the bare version.

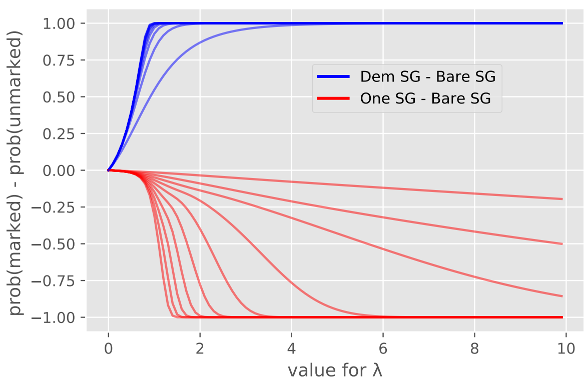

Figure 3: The system converges rapidly (and with speakers of low sophistication/rationality) towards the exclusive use of the demonstrative in case of the choice demonstrative vs. bare N for SIS; it converges slowly (and with speakers of higher sophistication/rationality) towards the bare form for ENS.

4 Conclusion

In this paper, I have developed a full version of the game of same and different, and I have applied it to a case of a language without grammaticalized articles. The system predicts that the grammaticalization of definite articles is simply the effect of standard (scalar) implicatures (once one demonstrative determiner is established as an alternative), while the grammaticalization of indefinite articles has to be explained by other parameters; it has been argued that standard scalar and manner implicatures should impede (or at least, slow down) the emergence of an indefinite article.

Acknowledgments

All calculations have been performed with Python 3, and diagrams have been made with *matplotlib*.

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