Representations of Magnitude and Comparison

This paper proposes that positive (1) and comparative (4) adjectives differ semantically in a fundamental respect: positives involve *comparison of analog magnitudes* and comparatives involve *comparison of exact numbers*.

(1) John is tall ∅/for a jockey/compared to Bill.

(2) John is taller than Bill.

In English, the presence or absence of the comparative morpheme signifies this difference.

Typical analyses treat positives as hidden comparatives; thus, both are analyzed as exact number comparisons. (For instance, von Stechow 1984; Heim 1985; Kennedy 1999) In these *degree*-based systems, gradable adjectives associate individuals with numerical values that represent the extent to which that individual possesses the relevant gradable property. These *degrees* are completely and densely ordered on *scales*. (Cresswell 1977) For instance, there are no important differences between the semantic representation of the positive in (3) and the comparative in (4), aside from how the standard degree, *c* in (3) and *d'* in (4), is calculated.

(3) ||John is tall|| =1 iff ∃d [john is d-tall and **d > c**] *c = salient standard height*

(4) ||John is taller than Bill|| =1 iff ∃d∃d' [john is d-tall & Bill is d'-tall & **d > d'**]

But positives differ empirically from comparatives in at least three ways, each of which indicate that this similar treatment is a mistake. First, comparatives allow measure phrases to describe the difference between the two objects compared, while positives do not.

(5) John is **three inches** taller than Bill.

(6) John is taller than Bill **by three inches**.

(7) *John is **three inches** tall for a jockey/compared to Bill.

(8) *John is tall for a jockey/compared to Bill **by three inches**.

This is unexpected if both constructions are simply *greater-than* relations between two degrees; both should allow modification by a measure phrase equally well. Second, comparatives allow crisp judgments, while positives do not. (Kennedy 2005)

(9) Context: a 100 page novel and a 99 page novel (from Kennedy 2005)
   a. This novel is longer than that one.
   b. #This novel is long compared to that one.

(10) Context: a boy is 5' 1/8", while everyone else in his family is exactly 5' tall
   a. This boy is taller than everyone else in his family
   b. #This boy is tall for a member of his family.
   c. #This boy is tall for a member of a family where everyone is exactly five feet tall.

Comparatives can describe even minute differences, but small differences are not enough to warrant application of the positive predicates in (9b) and (10). Again, this is a surprise if both positives and comparatives are *greater-than* relations between degrees because ">" is crisp *by definition*. Third, positive adjectives, even when accompanied by an explicit standard, are vague in a way that comparatives are not. This can be shown by whether a sentence like the universally quantified premise of a Sorites paradox compels us to accept or reject it as possibly true. Positive adjectives *even when they are accompanied by explicit standard* create compelling Sorites premises (11)-(13) (Fara 2000), but comparative adjectives (14) do not.

(11) √ Anyone who is one nanometer shorter than someone who is tall is also tall.

(12) √ Anyone who is one nanometer shorter than someone who is tall for a jockey is also tall for a jockey.

(13) √ Anyone who is one nanometer shorter than someone who is tall compared to Bill is also tall compared to Bill.

(14) # Anyone who is one nanometer shorter than someone who is taller than Bill is also taller than Bill.

Again this is surprising only if both positives and comparatives are both *greater-than* relations between two numbers. In (14), a one nanometer difference could potentially cross the boundary between those that are *taller than Bill* and those that are *not*. But in (11)-(13), a nanometer difference does not matter because the boundary is not clearly marked (despite providing as much information about the standard as in comparative). However, these properties of the positive can be accounted for using analog magnitudes.

Analog magnitude models accurately describe the abilities of many species (including humans) to mentally represent 'number' without counting. (see Gallistel & Gelman 1992, 2004; Dehaene, Dehaene-Lambertz & Cohen 1998) Analog magnitudes are inherently noisy mental representations that obey Weber's law, meaning that their imprecision linearly increases with their magnitude. (Gescheider 1997) The higher the number, the fuzzier it gets, and the harder it is to discriminate it from other high (and fuzzy) numbers. Humans (and rats) are very good at discriminating quantities of 2 and 3, but they get worse when forced to discriminate 8 from 9, 22 from 26, or 360 from 380. Analog magnitudes are Gaussian curves, which can be added, subtracted or compared. So, a positive adjectival construction can be a comparison, but not of two exact degrees.

(15) Positive Adjectives: $\Delta \succ \Delta'$ *where Δ, Δ' are analog magnitudes and $\succ$ is defined over analog magnitudes*

This accounts for the differences outlined above. First, measure phrases can describe the distance between two degrees on a scale, but analog magnitudes do not exist on a scale, nor are their values precise enough to measure differences. Second, because analog magnitudes are only approximate values that can be difficult to discriminate, we cannot make crisp judgments with the $\succ$ relation. This predicts that crispness is a function of discriminability. This prediction is borne out. A difference of one foot is enough for a crisp judgment when comparing 5 and 6 feet but not 200 and 201 feet.

(16) Context: John = 6', Bill = 5'; John's office building = 200' Bill's building = 201'
- a. John is tall compared to Bill.
- b. #?John's office building is tall compared to Bill's office building.

Third, positive adjectives are vague because Gaussian values do not have boundaries in the sense that degrees (or degree intervals, which have precise boundaries) do. It is easy to overlook the boundary of an analog magnitude because it simply doesn't exist.

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