LINEARIZING SETS: *EACH OTHER*

Alda Mari
CNRS/ENST mari@enst.fr

1 A unified account for *each other*. The problem of linear orderings

Asymmetric configurations (1) represent a major hurdle for any existing attempt to provide a unified account of reciprocal sentences (e.g., among many others, Fiengo and Lasnik, 1973; Langendoen, 1978; Schwarzschild, 1996; Saulerland, 1998; Dalrymple et al., 1998.). This paper is a further attempt in this direction.

(1) The tables are stacked on top of each other

The difficulty is grounded in the impossibility of extending weak (2b) reciprocity to such cases, let alone strong reciprocity (2a).

(2) The boys are looking at each other (Let $\|$ the boys $\|^{M,g}$ = $\{\{j\},\{r\},\{m\}\}$ (i.e., plurality P))

a. Strong rec.: $\forall x \subset P\ \forall y \subset P\ (xRy)$ e.g., $\forall x \subset \{\{j\},\{r\},\{m\}\}\ \forall y \subset \{\{j\},\{r\},\{m\}\}$ (x look at y)

b. Weak rec.: $\forall x \subset P\ \exists y \subset P\ (xRy)$ e.g., $\forall x \subset \{\{j\},\{r\},\{m\}\}\ \exists y \subset \{\{j\},\{r\},\{m\}\}$ (x look at y)

Weak reciprocity cannot capture linear orderings insofar as the last element has no other element with which it can stand in the relation provided by the predicate (e.g., for (1), the table at the bottom of the pile has no table beneath it). It has been argued that linear orderings are a case of weak reciprocity for which the context is benevolent and allows us to not consider all the elements (Fiengo and Lasnik, 1973; Sauerland, 1998). Others have suggested that in this case, the context can provide enough information to retrieve a weak reciprocity schema (e.g., Schwarzschild, 1996).

It has, however, also been convincingly argued that linear orderings are a common configuration that must receive a semantic account (Dalrymple et al., 1998). A proper representation can be provided under the assumption that relation R (e.g., *be on top of*) can be analyzed into two converse relations, R^+ and R^-. One refers to R^+-type and R^--type entities if the entities satisfy R^+ or R^- respectively.

(3) $\forall x \subset P\ ((R^+(x)) \rightarrow \exists y \subset P\ ((R^-(y))\ (R(x,y))))$

For (1), (3) states that for every table that is of type R^+, there is a table of type R^-, i.e., for each table that is on top, there is a table that is beneath it.

At this point, different accounts for *each other* should be admitted. The reason why (3) cannot be generalized to the other cases is that it predicts that, whenever the plurality is composed of only two elements, the relation need not be symmetrical. For (2), if $\|\ boys\ \|^{M,g'}$ = $\{\{j\},\{r\}\}$, it predicts that the sentence is interpretable if and only if John is of type R^+ and Robert is of type R^-, i.e., if and only if John looks at Robert, but Robert does not look at John.

This leads to isolating the locus of the incompatibility between strong/weak reciprocity and linear orderings in the following correlation: whenever there are only two elements, R is mandatorily symmetric, and this cannot be the case when the predicate is asymmetric. Note that (1) cannot be interpreted if the plurality is composed of only two tables.

This paper provides a solution to this problem and presents a unified account for *each other*. Two major predictions are made: for a plurality of two elements, strong reciprocity is mandatory, and all linear orderings are compatible with the reciprocal relation with the exception of comparatives (in simple sentences):

(4) *My relatives are taller than each other

2 The syntax and semantics of *each other*

Due to the semantics of $other$, *each other* has been thought of as denoting the function that picks, for each subplurality x in a given cover, that particular subplurality y in the same cover, different from x, that stands in relation R to x (Bennett, 1974, Heim et al., 1981). This requires that *each other* be doubly indexed: once to the plurality denoted by its antecedent taken collectively, and once to the same plurality taken distributively. Since (Schwarzchild, 1996), it is assumed that the translation of the reciprocal contains two variables, the latter bound by the distributivity operator introduced by the VP (5). The semantic account follows. Let d be the set of entities, and $\mathcal{C}(d)$ the set of the covers of d. For $g(x_i) = C \in \mathcal{C}(d)$, for every subplurality $x_k \in g(x_i)$ the function *Each Other* (= g(*each other*)) gives a subplurality different from x_k that stands in relation R to it. For (2), assuming C = $\{\{j\},\{r\}\}$, (6) is obtained:

(5) (John and Robert)$_i$ $Part_k$ look at each other$(x_i)(x_k)$

(6) *Each Other*(x_k) = $\{r\}$ if x_k = $\{j\}$ and $\{j\}$ if x_k = $\{r\}$.

In our account, we assume that *each other* is only indexed to its antecedent, and recover the $other$-meaning semantically, from the form of the image the function *Each Other*. We consider that the argument of the function is not a particular subplurality but the set itself. Consequently, the value of the function is not a particular subplurality, but a sequence, i.e., a linearization of the set. Let d be the set and p the size of d:

(7) *Each Other*(d) = $[x_1, x_2, ..., x_n]$ s.t:

i. the size of the sequence is ≥ 3 entity-types,
ii. for $1 \leq i \leq n-1$, x_i is of type R^+ and x_{i+1} of type R^- (hence x_1 is of type R^+ and x_n of type R^-),
iii. the majority of the members has to be involved in R, and p is possibly the cardinality of this majority (Schwarzschild, 1996).

Every two subsequent x_i, x_{i+1} form a pair that stands in relation R (the members of each pair standing in $other$ relation). The order of the members in the sequence is random. Every reciprocal configuration can be represented by a sequence defined as in (7). In particular, each member appears: (a) for strong reciprocity, $n-1$ times (hence $n \gg p$); (b) for weak reciprocity, at least once (hence $n \geq p$); (c) for linear orderings, exactly once (hence $n = p$). The cardinality of the majority (6iii) is contextually determined.

3 The nature of the sequence: predictions

Symmetric and asymmetric predicates. This account captures straightforwardly the fact that whenever there are only two elements, strong reciprocity is mandatory and, for linear order interpretation, three members are needed. Let P be the plurality composed of John and Robert and whose cover is $\{\{j\},\{r\}\}$. According to (7), we obtain: *each other* $(\{\{j\},\{r\}\})$ = [j,r,j] where the entities to the left of the commas are of type R^+, and those to the right of type R^-. The sequence [j,r,j] is well-formed since r is of both types R^+ and R^- and the sequence contains three different entity-types. For linear order denoting predicates, the requirement (7ii) can only be satisfied if the plurality is composed of three members, since a given object cannot be, for instance, both on top and bottom. The resulting schema will be: *each other* $\{\{a\},\{b\},\{c\}\}$ = [a,b,c].

Comparatives. The sequence is random and all permutations of entities are admitted, provided that the constraints (7i-iii) are satisfied. It follows that the entities are seen as interchangeable (for a similar notion, see Keenan, 1987). For comparatives, since the positions of the elements in the sequence are fixed by the properties of the denotations, no permutation is possible. They might be compatible with reciprocal interpretation if the context provides the possibility of a permutation:

(8) They look alternately taller than each other in different scenes (http://p081.ezboard.com)

Partitions. Lexical or contextual factors can impose partitioning (*the men and the women are married to each other*; the couples are the subpluralities of the partition). The following rules apply, for the interpretation of reciprocal sentences, in the following order:

1. **General rule**. *Each other* takes set d as its argument and gives a sequence that satisfies (7).
2. **Partition rule**. When required, the set can be partitioned in such a way that each partition satisfies (7).

It has to be emphasized that, in the second case, *each other* does not associate each subplurality with a different subplurality, but, as for the entire set, it gives the linearization of each subplurality. These rules are ordered by preference. Whenever it is possible, a configuration in which the partitions overlap is preferable. This preference is explained by the fact that *each other* corefers with the set taken collectively, and the existence of a unique sequence captures the fact that all the members of the plurality are involved in a unique event. This preference explains why state verbs such as *know* are associated with strong reciprocity. Actions can dynamically evolve and might reach a strong reciprocal configuration (in this case the partitioning is temporal). States cannot, and when this is possible, the configuration with the highest possible integration among the members is selected.

Geometry. A concern might be raised with respect to spatial relations that, in some border cases - especially with relatively big objects such as tables, - allow pairwise, asymmetric configurations, countering (7i). For example, four tables could be arranged in two piles of two, and sentence (1) would still be interpretable. Obviously such a configuration is incompatible with actions (two boys hitting each a different boy). Spatial arrangements seem to behave differently, since any object of the plurality could have equally occupied any position; in particular, the tables could have been disposed in a unique pile without being fundamentally differently affected (condition of permutability). The mathematical object (sequence) can be geometrically realized in different ways.

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